Comparisons of Simple Step and Dynamic Step Intervention Models

Supplement to "Power Computations for Interventions Analysis" by A.I.McLeod and E.R.Vinglis

Summary

Graphical and numerical comparisons illustrate the difference between a dynamic step intervention and a simple step intervention adjusted so both models have the same steady-state gain. The asymptotic theoretical power in each model with AR(1) noise is computed for a 5% two-sided test that the gain is zero. The dynamic model is simulated and fit using a simple step intervention and the empirical power of a two sided test of $H_0 : \omega_0 = 0$ is determined. It is found that the simple step intervention provides a reasonable approximation for estimating the power when $\phi_1 \leq 0.5$.

Models

The dynamic step intervention model with AR(1) noise may be written,

$$z_t = \xi + \omega_0 / (1 - \delta_1 B) S_t^{(T)} + a_t / (1 - \phi_1 B)$$

where $a_t \sim \mathrm{NID}(0, \sigma_a^2)$.

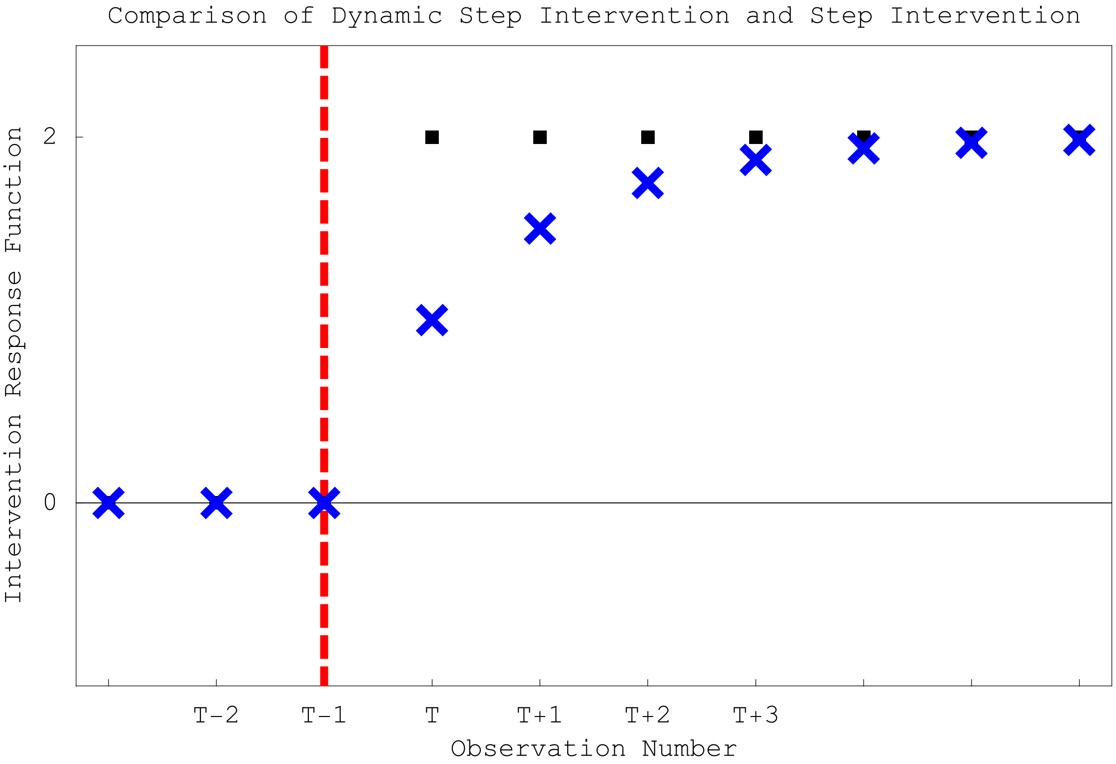
In this model the steady-state gain, which measures the long-run change, is $g = \omega_0 / (1 - \delta_1)$. We are interested in comparing this dynamic intervention model with the simple step intervention model,

$$z_t = \xi + \omega_0 S_t^{(T)} + a_t / (1 - \phi_1 B)$$

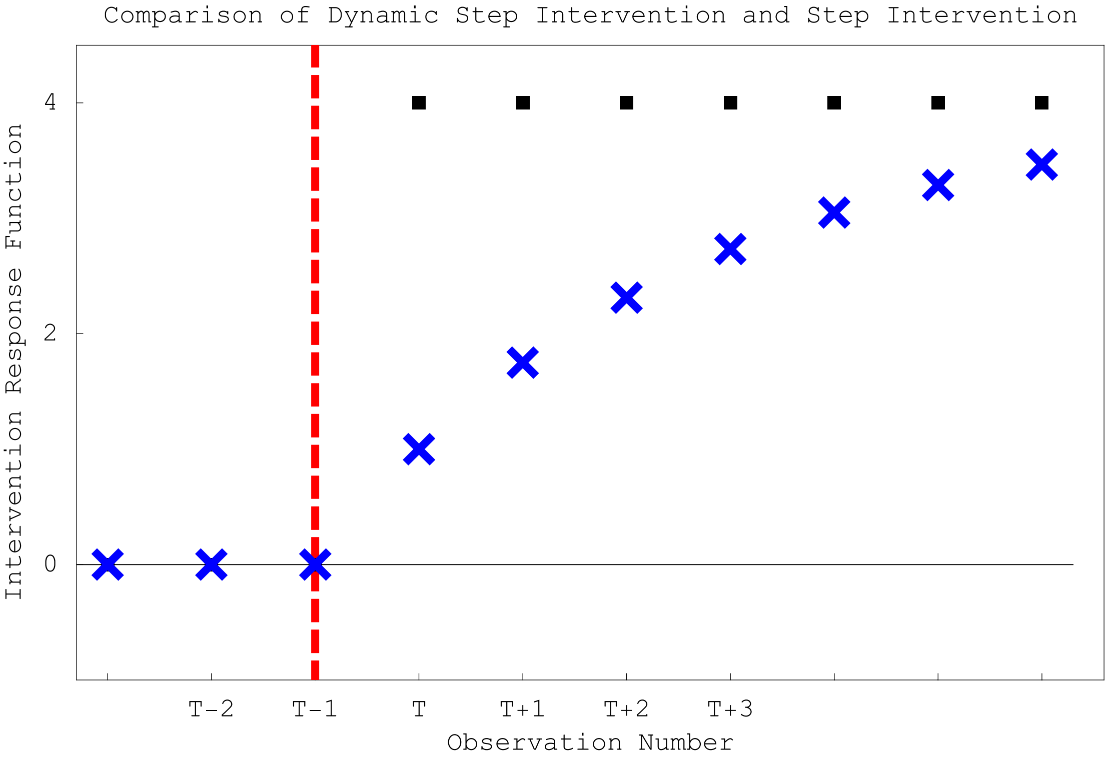
where we take $\omega_0 = g$.

Graphical Comparisons

Just concentrating on the intervention response the following plot shows illustrates the situation when $\omega_0 = 1$ and $\delta = 0.5$. The blue crosses show the response for the dynamic intervention $1 / (1 - 0.5 B) S_t^{(T)}$ and $2 S_t^{(T)}$



And when $\omega_0 = 1$ and $\delta = 0.75$, we obtain,



▼

Tables and Results of Simulation Experiments

The triplets show the power of a two-sided test at the 5% level of the null hypothesis that the gain is zero. The first entry in each triplet is the asymptotic theoretical power for the dynamic model and the second entry is the corresponding asymptotic power in simple step intervention. The third entry is the empirical power based on 1000 simulations when the dynamic model is simulated but the simple step intervention model with AR(1) error is fit. In the third case the null hypothesis, that the gain is zero, is $H_0 : \omega_0 = 0$.

$\phi = -0.75$

<table>
<tr><th></th><th>$\omega_0 = 0.5$</th><th>$\omega_0 = 0.75$</th><th>$\omega_0 = 1.$</th></tr>
<tr><td>$\delta = 0.25$</td><td>0.979|0.252|0.977</td><td>1.000|0.490|0.998</td><td>1.000|0.972|0.999</td></tr>
<tr><td>$\delta = 0.5$</td><td>1.000|0.490|0.999</td><td>1.000|0.827|0.999</td><td>1.000|1.000|0.999</td></tr>
<tr><td>$\delta = 0.75$</td><td>1.000|0.732|0.999</td><td>1.000|0.972|0.999</td><td>1.000|1.000|0.999</td></tr>
</table>

$\phi = -0.5$

<table>
<tr><th></th><th>$\omega_0 = 0.5$</th><th>$\omega_0 = 0.75$</th><th>$\omega_0 = 1.$</th></tr>
<tr><td>$\delta = 0.25$</td><td>0.928|0.252|0.930</td><td>0.999|0.490|0.995</td><td>1.000|0.972|0.999</td></tr>
<tr><td>$\delta = 0.5$</td><td>0.999|0.490|0.998</td><td>1.000|0.827|0.999</td><td>1.000|1.000|0.999</td></tr>
<tr><td>$\delta = 0.75$</td><td>1.000|0.732|0.999</td><td>1.000|0.972|0.999</td><td>1.000|1.000|0.999</td></tr>
</table>

$\phi = 0$

<table>
<tr><th></th><th>$\omega_0 = 0.5$</th><th>$\omega_0 = 0.75$</th><th>$\omega_0 = 1.$</th></tr>
<tr><td>$\delta = 0.25$</td><td>0.635|0.252|0.648</td><td>0.921|0.490|0.924</td><td>1.000|0.972|0.999</td></tr>
<tr><td>$\delta = 0.5$</td><td>0.933|0.490|0.931</td><td>0.999|0.827|0.997</td><td>1.000|1.000|0.999</td></tr>
<tr><td>$\delta = 0.75$</td><td>0.996|0.732|0.998</td><td>1.000|0.972|0.999</td><td>1.000|1.000|0.998</td></tr>
</table>

$\phi = 0.25$

<table>
<tr><th></th><th>$\omega_0 = 0.5$</th><th>$\omega_0 = 0.75$</th><th>$\omega_0 = 1.$</th></tr>
<tr><td>$\delta = 0.25$</td><td>0.417|0.252|0.421</td><td>0.724|0.490|0.758</td><td>0.996|0.972|0.993</td></tr>
<tr><td>$\delta = 0.5$</td><td>0.747|0.490|0.745</td><td>0.969|0.827|0.966</td><td>1.000|1.000|0.997</td></tr>
<tr><td>$\delta = 0.75$</td><td>0.938|0.732|0.951</td><td>0.999|0.972|0.998</td><td>1.000|1.000|0.995</td></tr>
</table>

$\phi = 0.5$

	ω_0 = 0.5	ω_0 = 0.75	ω_0 = 1.
δ = 0.25	0.226|0.252|0.241	0.416|0.490|0.466	0.879|0.972|0.880
δ = 0.5	0.439|0.490|0.445	0.745|0.827|0.758	0.997|1.000|0.974
δ = 0.75	0.673|0.732|0.692	0.937|0.972|0.932	1.000|1.000|0.955

ϕ = 0.75

	ω_0 = 0.5	ω_0 = 0.75	ω_0 = 1.
δ = 0.25	0.106|0.252|0.140	0.161|0.490|0.232	0.400|0.972|0.367
δ = 0.5	0.178|0.490|0.226	0.304|0.827|0.328	0.725|1.000|0.586
δ = 0.75	0.280|0.732|0.326	0.487|0.972|0.525	0.927|1.000|0.698

ϕ = 0.9

	ω_0 = 0.5	ω_0 = 0.75	ω_0 = 1.
δ = 0.25	0.072|0.252|0.121	0.086|0.490|0.160	0.147|0.972|0.157
δ = 0.5	0.101|0.490|0.189	0.133|0.827|0.201	0.272|1.000|0.252
δ = 0.75	0.142|0.732|0.238	0.199|0.972|0.283	0.438|1.000|0.335