K-Fold Cross-Validation

Primary method for estimating a tuning parameter λ (such as subset size)

• Divide the data into K roughly equal parts

1	2	3	4	5

Validation	Train	Train	Train	Train	
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for each k = 1, 2, ... K, fit the model with parameter λ to the other K - 1 parts, giving β^{-k}(λ) and compute its error in predicting the kth part:

$$E_k(\lambda) = \sum_{i \in kth \ part} (y_i - \mathbf{x}_i \hat{\beta}^{-k}(\lambda))^2.$$

This gives the cross-validation error

$$CV(\lambda) = \frac{1}{K} \sum_{k=1}^{K} E_k(\lambda)$$

• do this for many values of λ and choose the value of λ that makes $CV(\lambda)$ smallest.

Typically we use K = 5 or 10.

Cross-validation- revisited

Consider a simple classifier for wide data:

- Starting with 5000 predictors and 50 samples, find the 100 predictors having the largest correlation with the class labels
- Conduct nearest-centroid classification using only these 100 genes

How do we estimate the test set performance of this classifier?

Apply cross-validation in step 2?

This is WRONG!

- It ignores the fact that the procedure has already "seen" the labels of the training data, and made use of them. This is a form of training and must be included in the validation process.
- It is easy to simulate realistic data with the class labels independent of the outcome, so that
 - true test error =50%, but
 - "Wrong" CV error estimate is zero!
- We have seen this error made in 4 high profile microarray papers in the last couple of years. See Ambroise & McLachlan (2002) .

The Wrong and Right Way

- \times Wrong: Apply cross-validation in step 2.
- \checkmark *Right:* Apply cross-validation to steps 1 and 2.



