Mathematics of Financial Options

Chapter 7 — Bond Pricing with Default

Dr. Hristo Sendov

Department of Statistical and Actuarial Sciences

Winter 2019

Introduction

So far we have discussed loans, such as a car loan or a mortgage, in which an amount is borrowed and then repaid in equal payments.

Such repayment structures are very common in personal finance applications.

We turn attention to a different way to borrow money common in government and corporate debt markets.

Corporate and Government Bonds
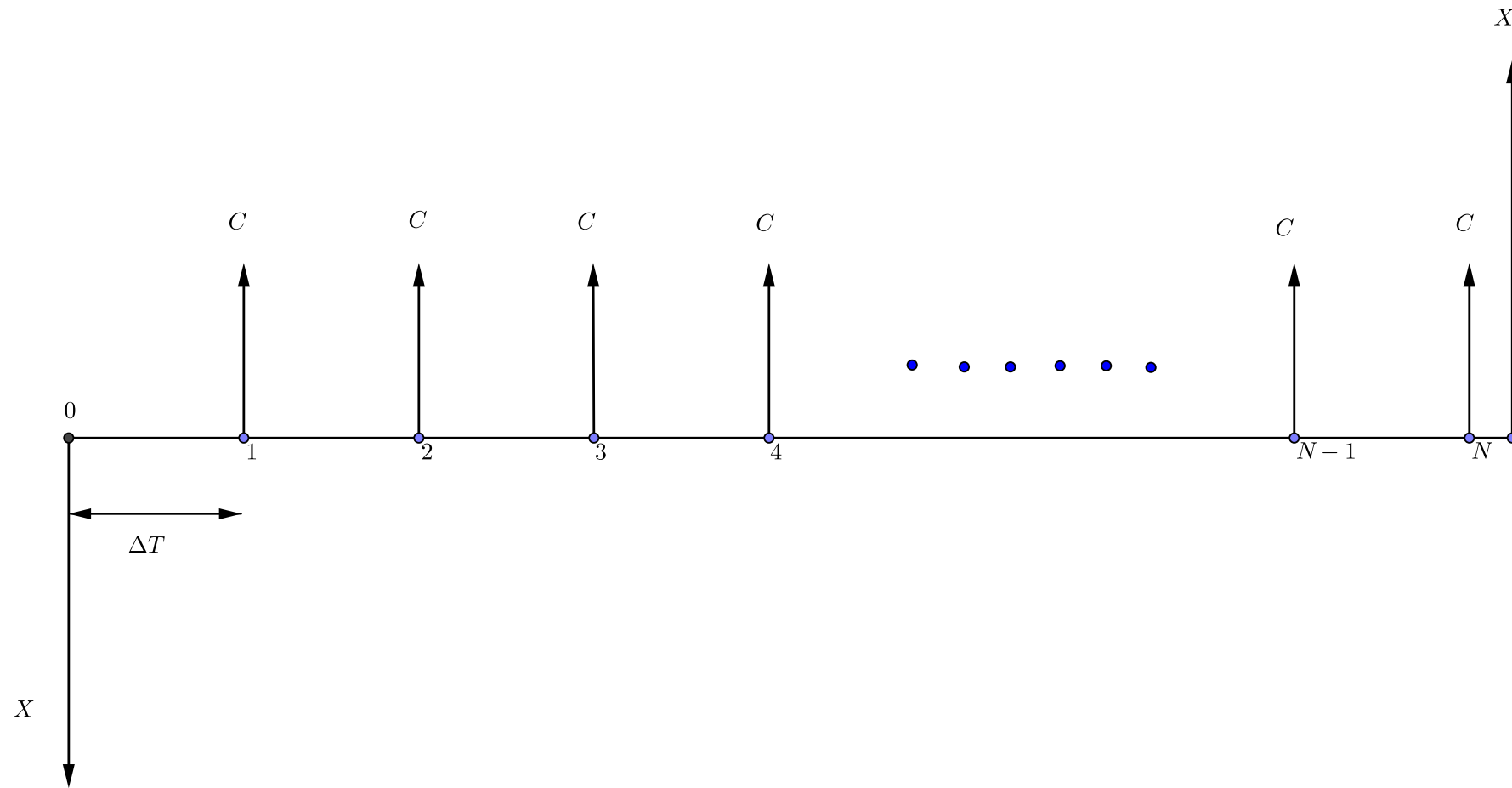
At time 0 the lender gives the borrower a "principal" amount of $X

At time periods 1 through N - 1 the borrower repays equal "coupons" $C.

At the final repayment period N the lender repays not just a coupon of $C but also the initial principal $X

Government Bonds

When governments of first-world countries (such as Canada, US, Germany, UK,…) borrow money, we assume that they are certain to repay their debts fully

That is because they tend to borrow in their own currency, the printing of which they control

Risk-free interest rate, is the rate of interest a lender would be willing to accept from a government borrower

If the borrower is sure to repay the loan, and the interest rate is r% per period, then the coupon $C has to be $rX

Government Bonds

It should be noted that the risk-free interest rate may differ depending on the time taken to repay the loan

This is called the **term structure of interest rates**

The term structure of interest rates fluctuates from day to day

Assume that the risk-free interest rate is a known, fixed, constant

Corporate Bonds

When companies and individuals borrow money there is always the chance they will be unable to repay it

Such an inability to repay is called a **default**

If the borrower is an individual or a company, the rate of return on the bond should be higher than the risk-free rate in order to compensate for the possibility of default of the individual or the company

Setting up the problem

- At time $k = 0$ the borrower receives \$X, the principal

- At times $k = 1,\ldots,$N-1 the borrower repays \$C, the coupon

- At time $k = N$ the borrower repays \$C + \$X

- The risk-free rate of interest is r% per period

- The probability of default in each time period is p

- What should \$C be?

Assumptions

- The event of default in each period is independent from the other periods

- Borrowers only default the instant before their next payment is due

- In the event of default, lenders receive a fraction of what is owed them $R(C+X)$, where R is a known constant

- The constant R is known as the **recovery rate**

Determining recovery rates

What do creditors get when a company goes bankrupt?

In Canada this is governed by the Company Creditor Arrangement Act

In the United States by the so-called Chapter 11 bankruptcy law

In a bankruptcy situation equity and preferred shareholders get nothing

Bond holders recover some money, but only after a long court process

Fighting court battles is not the business of many bond-holding entities such as pension funds

Defaulted bonds are sold to Vulture funds

Which usually pay recovery rates of about 40%

The Vulture funds fight the court battles

Table of Seniorities

Seniority	Security Type	Payments
Lowest	Equity (common stocks)	Dividends at management discretion
	Preferred shares	Dividends at pre-arranged rate
	Junior debt	High coupons
Highest	Senior debt	Lower coupons

Determining the probability of default

Default probabilities are estimated empirically

Of all the companies that have ever existed, look into those that are like the one I am analyzing, and of this group, how many defaulted

Done by rating agencies: Moody's, Duff & Phelps, Fitch, and Standard & Poor in the US and the Dominion Bond Rating Service and the Canadian Bond Rating Service in Canada

Difficulty: to decide when two companies are similar

Done by analysis of financial statements, knowledge of industry and experience

The rule of thumb 40% recovery rate of investment grade debt has default probabilities between 0.1% and 1% per year

THE END