Statistics 3858b Assignment 3

Handout March 1, 2017; Due date: March (TBA) , 2017

These problems are all from the course text unless otherwise stated. Not all questions will be marked.

1. 9.11 : 4, 19 (b) - (d) , 25, 26, 30 a, b,
   37 (Hint : the data are counts in 12 multinomial categories. What is the null hypothesis in terms of the parameter for this multinomial? The alternative is the complement.)
2. 11.6 : 11, 21 (part e means parametric bootstrap), 33
3. 13.8 : 16

4. The most common form of a test statistic of $H_0 : \theta = \theta_0$ versus $H_A : \theta \neq \theta_0$ is of the form

$$W = \frac{\hat{\theta} - \theta_0}{\sqrt{\hat{\text{Var}}(\hat{\theta})}}$$

where the denominator is a consistent estimator of the standard deviation of the estimator $\hat{\theta}$. The rejection region is then of the form reject if $W < c_L$ or $W > c_U$ for appropriate lower and upper critical values.

When there is a normal approximation, and sometimes with an exact distribution, W is of the form

$$W = \frac{\sqrt{n}\left(\hat{\theta} - \theta_0\right)}{\tau_n}$$

where the denominator is an estimator of a scaled standard deviation.

There is one other common form that occurs in practice. For an example of this consider the case of iid Normal sampling with

$$H_0 : \sigma^2 = \sigma_0^2 \text{ versus } H_A : \sigma^2 \neq \sigma_0^2$$

For this use the test statistic

$$W = \frac{(n-1)S^2}{\sigma_0^2}$$

where S^2 is the sample variance r.v.

Use the size α rejection region : reject if $W < q_{\alpha/2}$ or $W > q_{1-\alpha/2}$ where $q_{\alpha/2}$ is the $\alpha/2$ quantile and $q_{1-\alpha/2}$ is the $1 - \alpha/2$ quantile.

(a) What is the distribution of W, under the assumption that the null hypothesis is true? Hint : See the text Section 6.3.

(b) Give the formula for the confidence interval for σ^2 based on W and this rejection region. Do this by solving for the possible σ_0^2, for which $H_0 : \sigma^2 = \sigma_0^2$, that are NOT REJECTED.

(c) For $n = 9$ give the size $\alpha = .05$ give the lower and upper critical values. Where, if anywhere, is the alternative used?

(d) For $n = 9$, the summary data are

$$\sum_{i=1}^{n} x_i = 37.78 \quad \sum_{i=1}^{n} x_i^2 = 190.95 \ .$$

Give the 95% confidence interval for σ^2.

(e) For this data above give the 95% confidence interval for σ.

Suggested Problems

1. 9.11 : 6, 21, 23, 26, 30 a - b, 40
2. 10.9 : 19 (deals with construction of QQ plots)
3. 11.6 : 6 , 7 , 33