

Statistics 3858b Assignment 3

Handout March 5, 2018; Due date: March (TBA) , 2018

These problems are all from the course text unless otherwise stated. Not all questions will be marked.

The due date will be chosen after the term test as students have a few terms in other courses during the next week or so.

1. 9.11 : 19 (b) - (d) , 25, 26, 30 a, b, 37 (Hint : the data are counts in 12 multinomial categories. What is the null hypothesis in terms of the parameter for this multinomial? The alternative is the complement.)
2. 11.6 : 11, 21 (part e means parametric bootstrap), 33
3. 13.8 : 16

4. The most common form of a test statistic of $H_0 : \theta = \theta_0$ versus $H_A : \theta \neq \theta_0$ is of the form

$$W = \frac{\hat{\theta} - \theta_0}{\sqrt{\widehat{\text{Var}}(\hat{\theta})}}$$

where the denominator is a consistent estimator of the standard deviation of the estimator $\hat{\theta}$. The rejection region is then of the form reject if $W < c_L$ or $W > c_U$ for appropriate lower and upper critical values.

When there is a normal approximation, and sometimes with an exact distribution, W is of the form

$$W = \frac{\sqrt{n}(\hat{\theta} - \theta_0)}{\tau_n}$$

where the denominator is an estimator of a scaled standard deviation.

There is one other common form that occurs in practice. For an example of this consider the case of iid Normal sampling with

$$H_0 : \sigma^2 = \sigma_0^2 \text{ versus } H_A : \sigma^2 \neq \sigma_0^2$$

For this use the test statistic

$$W = \frac{(n-1)S^2}{\sigma_0^2}$$

where S^2 is the sample variance r.v.

Use the size α rejection region : reject if $W < q_{\alpha/2}$ or $W > q_{1-\alpha/2}$ where $q_{\alpha/2}$ is the $\alpha/2$ quantile and $q_{1-\alpha/2}$ is the $1 - \alpha/2$ quantile.

- (a) What is the distribution of W ? Hint : See the text Section 6.3.
- (b) Give the formula for the confidence interval for σ^2 based on W and this rejection region. Do this by solving for the possible σ_0^2 , for which $H_0 : \sigma^2 = \sigma_0^2$, that are NOT REJECTED.
- (c) For $n = 9$ give the size $\alpha = .05$ upper and lower critical values.
- (d) For $n = 9$, the summary data are

$$\sum_{i=1}^n x_i = 37.78 \quad \sum_{i=1}^n x_i^2 = 190.95 .$$

Give the 95% confidence interval for σ^2 .

- (e) For this data above give the 95% confidence interval for σ .

Suggested Problems

1. 9.11 : 6, 21, 23, 26, 30 a - b, 40
2. 11.6 : 6 , 7 , 33 These questions deal with two sample hypothesis test questions.