FORMULAE SHEET

Some common pdf's and properties are given on this page. There are other common distributions from the course that the student is expected to know.

1. The bivariate normal pdf is

$$
\begin{aligned}
f(x,y) \quad = \quad & \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} \exp\left\{ -\frac{1}{2(1-\rho^2)} \left[\frac{(x-\mu_X)^2}{\sigma_X^2} \right.\right. \\
& \left.\left. -2\rho\frac{(x-\mu_X)(y-\mu_Y)}{\sigma_X\sigma_Y} + \frac{(y-\mu_Y)^2}{\sigma_Y^2} \right] \right\}
\end{aligned}
$$

2. The marginal distributions of the bivariate normal are $N(\mu_X, \sigma_X^2)$ and $N(\mu_Y, \sigma_Y^2)$.

3. If $Z \sim N(0,1)$, $Y \sim \chi^2_{(n)}$ and they are independent, then

$$\frac{Z}{\sqrt{Y/n}} \sim t_{(n)} \; .$$

4. If $X_1, \ldots, X_n$ are iid $N(\mu, \sigma^2)$ then

$$\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{\sigma^2} \sim \chi^2_{(n-1)}$$

5. Gamma function and properties:

   (a)
   $$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}dx$$

   (b) For $\alpha > 0$, $\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$.

   (c) For integers $n \geq 1$, $\Gamma(n) = (n-1)!$.

   (d) $\Gamma(1/2) = \sqrt{\pi}$.

6. Gamma(α, λ) density

$$f(x) = \frac{\lambda^\alpha x^{\alpha-1}}{\Gamma(\alpha)} e^{-\lambda x} \mathrm{I}(x > 0)$$

MGF :

$$M(t) = \left(\frac{\lambda}{\lambda - t} \right)^\alpha$$

$$\mathrm{E}(X) = \frac{\alpha}{\lambda}, \quad , \mathrm{Var}(X) = \frac{\alpha}{\lambda^2} \ .$$

The $\chi^2_{(k)}$ distribution is the same as the Gamma$(\frac{k}{2}, \frac{1}{2})$ distribution.

Exponential density, with parameter λ, is the Gamma with $\alpha = 1$ density.

7. The Beta(a, b) distribution has density

$$f(x) = \begin{cases} \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1} & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\mathrm{E}(X) = \frac{a}{a+b}$$

8. Geometric(θ). $X \sim$ Geometric(θ), the number of failures prior to the first success of iid Bernoulli trials has pmf

$$P(k; \theta) = (1-\theta)^k \theta \ , \ k = 0, 1, 2, \ldots$$

$$\mathrm{E}(X) = \frac{1}{\theta} - 1 \ , \ \mathrm{Var}(X) = \frac{1-\theta}{\theta^2}$$

Note there is an other form of the geometric distribution, one that give Y = number of trials till the first success. The relation $Y = X + 1$ holds.

9. Binomial pmf

$$P(k) = \binom{n}{k} (1-\theta)^{n-k} \theta^k \ , \ k = 0, \ldots, n$$

$$\mathrm{E}(X) = n\theta \ , \ \mathrm{Var}(X) = n\theta(1-\theta)$$

10. Poisson pmf

$$P(k) = \frac{\lambda^k e^{-\lambda}}{k!} \ , \ k = 0, 1, 2, \ldots$$