3858 Assign 1-Q4

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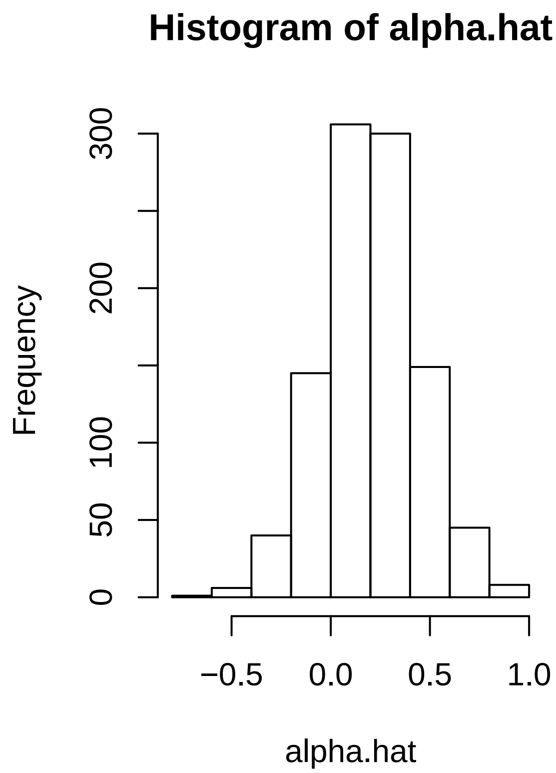
Having found the inverse function of cdf ($F^{-1}(u)$), to simulate a r.v. with distribution f, we can simulate a r.v U following $Unif(0,1)$ and then do the transformation $F^{-1}(U)$. By using the $Prop2.3DPage63$, $F^{-1}(U)$ can be regarded as the simulation of the distribution with cdf F.

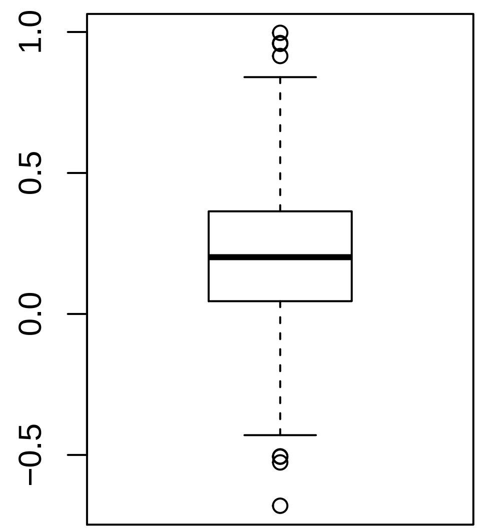
Since we assume the true value of $\alpha = 0.2$, here we only consider the inverse function when $\alpha \neq 0$.

```
#the inverse function, we set the actual value of alpha as the default value.
invcdf<-function(u,alpha = 0.2){
  (1/alpha)*(-1+sqrt((alpha-1)^2+4*alpha*u))
}

M=1000 # times of Monte Carlo replication
n=50 # the sample size
alpha.hat<-rep(0,M) #the vector used to store the estimate of alpha each time

for (m in 1:M){
  #simulate uniform r.v.
  U<-runif(n)
  #do the transform
  X<-invcdf(U)
  alpha.hat[m]=3*mean(X)
}
var(alpha.hat)
```

```
## [1] 0.05987488
```

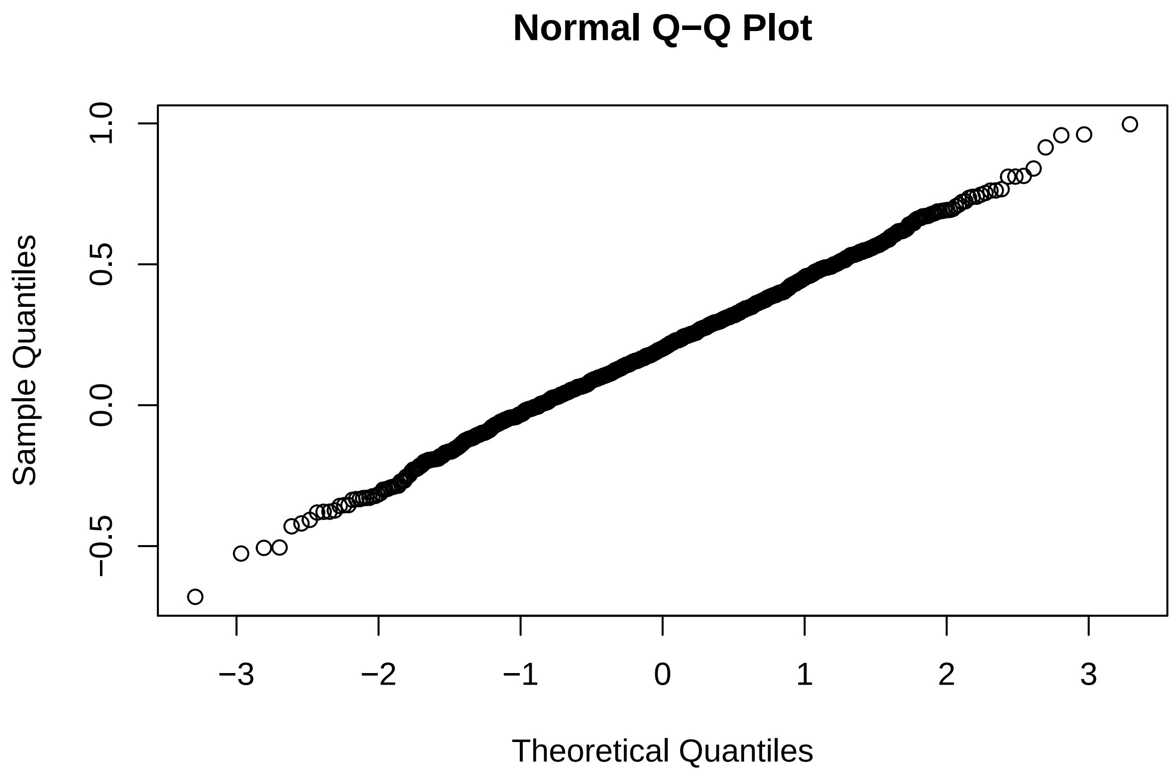
```
layout(matrix(1:2,1,2,byrow=TRUE))
hist(alpha.hat)
boxplot(alpha.hat)
```





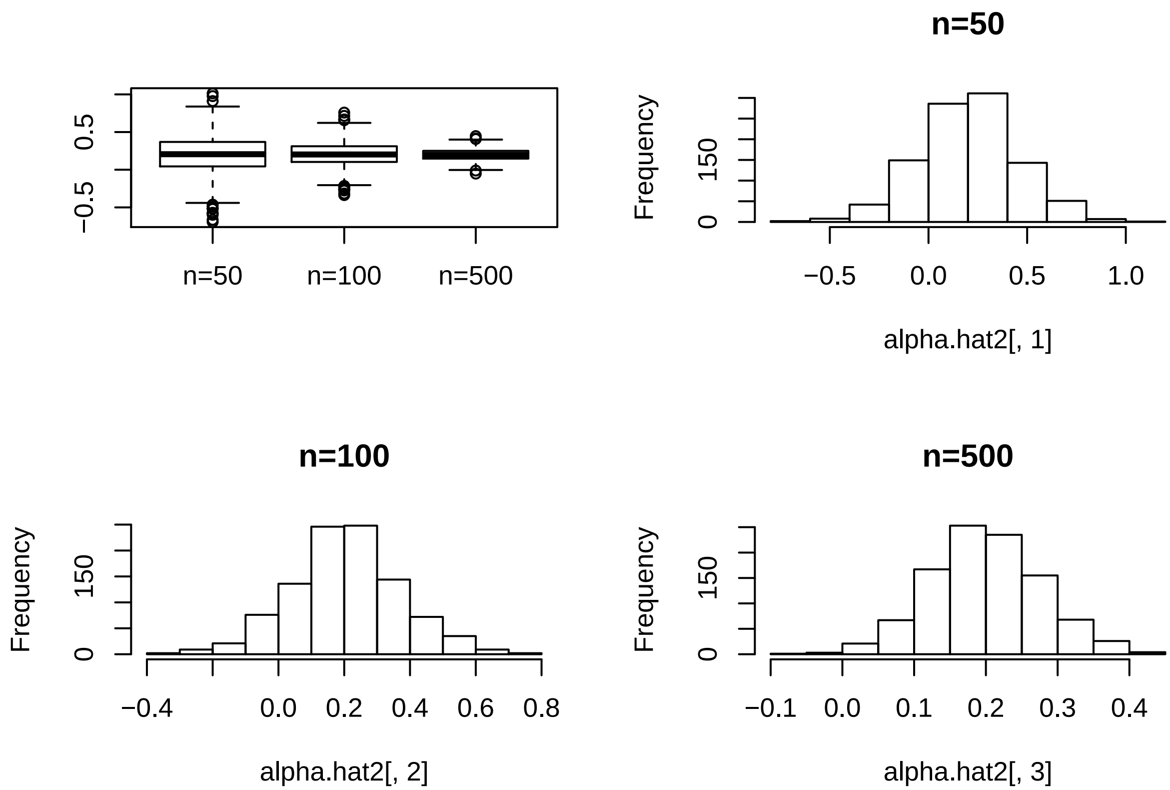
Then we use qqnorm to compare this with normal distribution.

```
#compare this with normal distribution
qqnorm(alpha.hat)
```



From the plot, the points are almost distributed on a straight line. Therefore, the true sampling distribution is very close to the normal distribution.

```r
alpha.hat2<-matrix(0,M,3,dimnames = list(NULL,c("n=50","n=100","n=500")))
M=1000
for (i in 1:3 ){
  a<-c(50,100,500)
  n<-a[i]
  for (m in 1:M){
  #simulate uniform r.v.
  U<-runif(n)
  #do the transform
  X<-invcdf(U)
  alpha.hat2[m,i]=3*mean(X)
}
}
layout(matrix(1:4,2,2,byrow=TRUE))
boxplot(alpha.hat2)
hist(alpha.hat2[,1],main="n=50")
hist(alpha.hat2[,2],main="n=100")
hist(alpha.hat2[,3],main="n=500")
```



From the boxplots and the histgrams, $\hat{\alpha}$ seems to be more close to the true value 0.2 as n increases.