University of Western Ontario
Statistics 3858b Term Test

March 18, 2009 5:30-6:30 PM

Instructor: R. J. Kulperger

Instructions:

I. Make sure that your name and ID number are the front of your exam booklet and question sheet. The exam questions are to be returned with the exam booklet.

II. All answers are to be written in the exam booklet, and only this will be graded.

III. The normal and chi-square tables from the Rice text is also attached.

1	2	3	TOTAL

Name : ______________________________

ID : ____________________

1. Suppose $X_1, X_2, \ldots, X_n$ are iid $N(\mu, \sigma^2)$ where $(\mu, \sigma^2) \in \Theta = R \times R^+$.

   a) Determine the MLE of (μ, σ^2).

   b) Give the sampling distribution of the MLE, stating any appropriate Theorems that may be useful for this.

c) Based on your result above give the formula for the $100(1-\alpha)\%$ confidence interval for σ^2.

d) A sample of size $n = 11$ is observed.
Some summary statistics for this data are $\bar{x}_n = 9.94$ and $\sum_{i=1}^{10}(x_i - \bar{x}_n)^2 = 223.83$
Give the observed values of the MLE.
Give the 90% confidence interval for σ^2 based on the MLE.

2. Suppose $X_1, \ldots X_n$ are iid Poisson, θ.

   a) Derive the MLE of θ. Calculate the Cramer-Rao lower bound. Does the MLE attain the Cramer-Rao lower bound? In one or two sentences discuss what this means in terms of the efficiency or other relevant properties of this estimator.

   b) In the rest of this question consider the null hypothesis $H_0 : \theta = \theta_0$ and the alternative hypothesis $H_A : \theta = \theta_1$ where $\theta_0 < \theta_1$.

      i) Derive the likelihood ratio test of the null hypothesis versus the alternative.

ii) By appropriate algebraic manipulations show the rejection region is of the form $\sum_{i=1}^{n} X_i$ (greater than or less than, which you will determine) a constant, say k.
Is this test (or equivalently rejection region) uniformly most powerful for all alternatives under the condition that $\theta_1 > \theta_0$, and explain.

3. Consider the following two distributions with the stated parameters

$$p(x|\theta_0) = \begin{cases} .2 & \text{if} \quad 0 \\ .3 & \text{if} \quad 1 \\ .3 & \text{if} \quad 2 \\ .2 & \text{if} \quad 3 \end{cases}$$

and

$$p(x|\theta_1) = \begin{cases} .1 & \text{if} \quad 0 \\ .4 & \text{if} \quad 1 \\ .1 & \text{if} \quad 2 \\ .4 & \text{if} \quad 3 \end{cases}$$

Suppose a sample of size 1 is taken with random data X. Consider the null hypothesis $H_0 : \theta = \theta_0$ and the alternative hypothesis $H_A : \theta = \theta_1$.

a) Why is the likelihood ratio test the best test of H_0 versus H_A? Explain by stating a relevant theorem or result from class, and then discussing in one or two sentences why this result is useful.

b) Give the random variable that is the likelihood ratio, that is give the likelihood ratio for every possible value X can take on.

c) Give the likelihood ratio test of size $\alpha = .2$.

d) For this likelihood ratio test of size .2 give the power of the test.