University of Western Ontario
Statistics 3858b Term Test

March 17, 2010 5:30-6:30 PM

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Instructions:

I. Make sure that your name and ID number are the front of your exam booklet and question sheet. The exam questions are to be returned with the exam booklet.

II. All answers are to be written in the exam booklet, and only this will be graded.

III. The normal and chi-square tables from the Rice text is also attached.

1	2	3	TOTAL

Name : ______________________________

ID : ____________________

1. a) Define Sufficient Statistic.

   b) State the Factorization Theorem.

   c) Suppose $X_1, X_2, \ldots, X_n$ are iid $N(\mu, \sigma^2)$ where $(\mu, \sigma^2) \in \Theta = R \times R^+$.

   Find a two dimensional sufficient statistic, and justify your answer using either (a) or (b) above.

d) In the setting of (c), derive the MLE of (μ, σ^2). Is the MLE a function of the sufficient statistic?

2. Suppose $X_1, \ldots X_n$ are iid exponential λ.

   a) Derive the MLE of λ.

   b) In the rest of this question consider the null hypothesis $H_0 : \lambda = \lambda_0$ and the alternative hypothesis $H_A : \lambda \neq \lambda_0$

      i) Derive the likelihood ratio test of the null hypothesis versus the alternative.

ii) State an appropriate Theorem, with conditions, that gives a limit distribution for $-2\log(\Lambda(X))$ where Λ is the generalized likelihood ratio.
Does this Theorem apply in the setting in the previous part (justify this in one or two sentences)?

iii) Suppose that $n = 50$, and that we observe $\sum_{i=1}^{n} x_i = 78.1$. Test the hypothesis $H_0 : \lambda = .7$ versus the alternative $H_0 : \lambda \neq .7$ at level .05. State your conclusion for this test.

3. An experiment is performed to determine is a coin is fair. In the experiment the coins are tossed 3 at a time. Each trial records the number of heads for these 3 coins, that is 0, 1, 2 or 3. This experiment of tossing coins in performed $n = 1000$ times, so that 3000 coin tosses were made.

   a) As a model for this experiment we have $X_i, i = 1, \ldots, n$ are iid Binomial(3, θ). Derive the MLE for θ. Using the data in the table in part (d) give the observed value of the MLE $\hat{\theta}$ for this data. This table gives the observed number of times X_i equals 0, 1, 2 or 3. Specifically the number of times the outcome of 0 observed heads out of the $n = 1000$ trials is 144.

b) The r.v $Y = (Y_0, Y_1, Y_2, Y_3)$ with Y_j = the number of times $X_i = j$, $i = 1, 2, \ldots, n$ in this experiment, that is the number of times the outcome j is observed, has a multinomial distribution on 4 categories.

If the coin tossing game really consists of 3 fair independent coin tosses for each of the $n = 1000$ trials, then Y will have a specified multinomial distribution. Otherwise it may have any arbitrary multinomial distribution on 4 categories.

State an appropriate null hypothesis and alternative to test if the coin tossing game is fair, in the sense that each trial is 3 iid fair coin tosses, versus it is not of this form?

c) Derive the form of $\Lambda(X)$ the generalized likelihood ratio test for the hypothesis test in (b).

d) As an approximation to the test statistic in (c), you may use Pearson's chi-squared statistic, or you may use $-2\log(\Lambda(X))$ you derived in (c). Test the hypothesis in (b) at level $\alpha = .05$ where the observed data is

number of heads	counts
0	144
1	379
2	354
3	123

State your conclusion for this hypothesis test.