Statistics 421b: Possible Projects

January 2005

The report on the project will include an appropriate literature review and referencing of material needed. Photocopies of the reference material may be requested by the instructor. Be sure to be careful in indicating which material comes from which source. Plagarism is a scholastic office. See the University Calendar. Electronic versions of data sets used are also to be submitted. In the case of a simulation study, the algorithm will be included in the report. An appendix will include the code, and an electronic version of the code will normally also be submitted.

The report will typically be about 15 to 20 pages, plus supporting appendices.

1. CAPM model report and review; portfolio pricing. This might be useful for economics students.

2. Simulations studies of any of the following :

   These can be done under an assumption of Geometric Brownian motion or some other price process model. In the case of a multivariate stock, as needed for portfolio problems or options on multiple stocks, you will need to choose an appropriate multidimensional price model.

   (a) Black Scholes pricing method and estimation based on data $S_{j\Delta}$, $j = 0, 1, ..., n$ and a study of the sampling distribution of pricing of European and Asian options. Effects of differing n. Are estimates biased? Does a similar pattern hold over many possible parameters?

   (b) Simulation of American option values; comparison with the binomial model. This would be useful for applied math students.

   (c) Simulation of the values for Asian options.

   (d) Simulation of values for Bermuda or other exotic options. Bermuda options can be on a single stock, or as a payout on several stocks with a payoff function of the form

   $$\mathrm{E}(\max((s_1 - K_1, s_2 - K_2, \ldots, s_n - K_n, 0))$$

   . The exercise times are at times $t_1 < t_1 < \ldots, t_m$. Pricing this for $n = 1$ or $n = 2$ would be sufficient.

(e) Any project implementing bond pricing.

(f) risk neutral pricing using an ARCH model and a study of sampling distribution of option prices

(g) commodities pricing; eg power prices. Why does risk neutral not work well here?

(h) review and comparison the various methods of simulation discussed in the literature; antithetic method

(i) Ito Taylor expansions and simulation of SDE systems. This is useful for students in probability or applied mathematics and are interested in simulation of SDE processes other than just geometric Brownian motion.

3. Numerical solutions of the boundary PDE system for American and European options. For example one could implement Rungge Kutta methods and other discretization methods and study the stability of such methods. For an Applied Math student this would be a natural follow up or application of their numerical PDE methods course.

4. Pricing of options for commodities. For example electricity options pricing. This was in the news for much of the fall and winter of 2000-2001 in California, and to a lesser extent in Alberta and Pennsylvania.

5. Any other reasonable project that a student can come up with can be used. This will be chosen with the approval of the instructor.

Remarks

For these simulation applications to evaluate option values, the project should address various aspects. For example suppose that one models a stock by geometric Brownian motion so that the stock value is

$$S_t = S_0 e^{(\mu - \sigma^2/2)t + \sigma W_t}$$

where W is standard Brownian motion. Under the risk neutral measure we have the same process except that μ is replaced by r, the risk free interest rate, that is

$$S_t = S_0 e^{(r - \sigma^2/2)t + \sigma W_t}$$

Thus for a contingent claim Y the value of an option with exercise time T is

$$V(s) = \mathrm{E}(Y e^{-rT})$$

where the expectation is with respect to the risk neutral measure or process. For example for a European call we have

$$Y = (S_T - K)^+$$

where K is the exercise price. For an Asian option we have Y of the form $Y = H(X)$ for some payoff function

$$X = \frac{1}{\tau} \int_{T-\tau}^{T} S_t dt$$

For an Asian call we have $H(x) = (x - K)^+$.

1. The value depends on various parameters σ, r, K, T. By simulating over a variety of parameters the student can comment on how the option values change as a particular parameter changes. For example what happens for a call type option as K increases. Are you simulations consistent with what you think should happen? There are many parameters that can be changed or varied.

2. You will need to comment on how to simulate a sample path of S_t (as needed for an Asian option). How many simulation replicates should you use?

3. Related to the first point how many parameter settings should you consider to answer questions about the effects of varying parameters?

4. What general values of parameters should you use? This will have to be related to parameters that come from fitting models to observed data, or choosing parameters that gives similar behaviour to what is observed, for example means and variances of the observed data.

One could also implement pricing based on binomial trees. What happens when the number of increments increases so the binomial tree in some sense converges to geometric Brownian motion? A study of the effects on approximating the value of an option, such as an Asian option option could be studied.

Report and Submission

Two copies of the report will be submitted. The instructor will evaluate one. Each student team will be given one other project to make a critical evaluation. The critical evaluation will address

- the description of the project and question undertaken
- the motivation or background - this will depend on which project is undertaken
- the relevant methodology used and the description of this methodology
- the implementation and description of results
- summary and comments made by the project authors
- All of the above may address both positive and negative points, along with justification that your evaluation may require.