University of Western Ontario
Statistics 3858b Mid-Term Test

February 11, 2009 5:30-6:20 PM

Instructor: R. J. Kulperger

Instructions:

I. Make sure that your name and ID number are the front of your exam booklet and question sheet. The exam questions are to be returned with the exam booklet.

II. All answers are to be written in the exam booklet, and only this will be graded.

III. The normal table from the Rice text is also attached.

1	
2	
3	
Total (50)	

Name : ________________________________

ID : ________________________________

Questions

1. Give the definition, and example if required, of the following:

   a) Define parameter space. In the case of the Gamma distribution family, give the parameter space for this family.

   b) Suppose the data for an experiment is a random of size n sample. What is a statistic in this setting? Give an two examples of a statistic for a parametric family of distributions of your choosing. Give an example of a random variable that is not a statistic.

c) What is meant by a parameter being *identifiable*?
For a Binomial(3, p) family of distributions, with $p \in \Theta = (0, 1)$, verify that p is identifiable.

d) Define consistent estimator.

2. Consider the regular MLE case, with pdf $f(x;\theta)$, $\theta \in \Theta$ and $X \sim f$. In each part state any assumptions used.

   a) Prove that

$$\mathrm{E}\left(\frac{\partial \log f(X;\theta)}{\partial \theta}\right) = 0$$

   b) Let $\mathrm{Var}\left(\frac{\partial \log f(X;\theta)}{\partial \theta}\right) = \mathrm{I}(\theta)$. Prove that

$$\mathrm{I}(\theta) = -\mathrm{E}\left(\frac{\partial^2 \log f(X;\theta)}{\partial \theta^2}\right)$$

3. Consider the family of distributions with density

$$f(x|\theta) = (\theta + 1)x^{\theta}, \quad 0 < x < 1$$

where $\theta \in \Theta$ is the parameter space.

Let X denotes a r.v. with this density.

a) Verify that any $\theta > 0$ yields a valid parameter, that is it produces a valid probability distribution. *Remark : This verifies that* $\Theta = (0, \infty)$ *is a appropriate parameter space.*

b) Suppose X_i, $i = 1, \ldots, n$ is a random sample of size n from this distribution. Give the likelihood and log likelihood function of θ.

c) Obtain the Maximum Likelihood Estimator (MLE) $\hat{\theta}$ of θ using your choice of one of the functions in part (b).

d) Obtain the method of moments estimator.

e) Calculate the information $I(\theta)$. Give the normal approximation to the distribution of $\hat{\theta}$.

f) Suppose that $n = 50$. For this data we observe $\sum_{i=1}^{n} x_i = 29.18$ and $\sum_{i=1}^{n} \log(x_i) = -37.46$. Find the observed value of the MLE and of the method of moments estimator, that is the estimates for this data.

Give the (observed) approximate 90% confidence interval for θ based on the MLE.