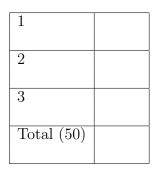
## University of Western Ontario Statistics 3858b Mid-Term Test

February 11, 2009 5:30-6:20 PM

Instructor: R. J. Kulperger

## Instructions:

- I. Make sure that your name and ID number are the front of your exam booklet and question sheet. The exam questions are to be returned with the exam booklet.
- II. All answers are to be written in the exam booklet, and only this will be graded.
- III. The normal table from the Rice text is also attached.



Name :	
ID :	

## Questions

- 1. Give the definition, and example if required, of the following:
  - a) Define parameter space. In the case of the Gamma distribution family, give the parameter space for this family.

b) Suppose the data for an experiment is a random of size n sample. What is a statistic in this setting? Give an two examples of a statistic for a parametric family of distributions of your choosing. Give an example of a random variable that is not a statistic. c) What is meant by a parameter being *identifiable*? For a Binomial(3, p) family of distributions, with  $p \in \Theta = (0, 1)$ , verify that p is identifiable.

d) Define consistent estimator.

- 2. Consider the regular MLE case, with pdf  $f(x; \theta), \theta \in \Theta$  and  $X \sim f$ . In each part state any assumptions used.
  - a) Prove that

$$\operatorname{E}\left(\frac{\partial \log f(X;\theta)}{\partial \theta}\right) = 0$$

b) Let 
$$\operatorname{Var}\left(\frac{\partial \log f(X;\theta)}{\partial \theta}\right) = I(\theta)$$
. Prove that  
$$I(\theta) = -E\left(\frac{\partial^2 \log f(X;\theta)}{\partial \theta^2}\right)$$

3. Consider the family of distributions with density

$$f(x|\theta) = (\theta + 1)x^{\theta}, \quad 0 < x < 1$$

where  $\theta \in \Theta$  is the parameter space.

Let X denotes a r.v. with this density.

a) Verify that any  $\theta > 0$  yields a valid parameter, that is it produces a valid probability distribution. *Remark* : This verifies that  $\Theta = (0, \infty)$  is a appropriate parameter space.

b) Suppose  $X_i$ , i = 1, ..., n is a random sample of size n from this distribution. Give the likelihood and log likelihood function of  $\theta$ .

c) Obtain the Maximum Likelihood Estimator (MLE)  $\hat{\theta}$  of  $\theta$  using your choice of one of the functions in part (b).

d) Obtain the method of moments estimator.

e) Calculate the information  $I(\theta)$ . Give the normal approximation to the distribution of  $\hat{\theta}$ .

f) Suppose that n = 50. For this data we observe  $\sum_{i=1}^{n} x_i = 29.18$ and  $\sum_{i=1}^{n} \log(x_i) = -37.46$ . Find the observed value of the MLE and of the method of moments estimator, that is the estimates for this data.

Give the (observed) approximate 90% confidence interval for  $\theta$  based on the MLE.