University of Western Ontario
Statistics 3858b Mid-Term Test

February 10, 2010 5:30-6:20 PM

Instructor: R. J. Kulperger

Instructions:

I. Make sure that your name and ID number are the front of your exam booklet and question sheet. The exam questions are to be returned with the exam booklet.

II. All answers are to be written in the exam booklet, and only this will be graded.

III. The normal table from the Rice text is also attached.

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2	
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Total (50)	

Name : ________________________________

ID : ________________________________

Questions

1. Give the definition, and example if required, of the following:

   a) Define parameter space. In the case of the geometric distribution family, give the parameter space for this family.

   b) For a statistical experiment define Statistic. Give an two examples of a statistic for a parametric family of distributions of your choosing. Give an example of a random variable that is not a statistic.

c) Define parameter identifiability.
For a Geometric(p) family of distributions, with $p \in \Theta = (0, 1)$, verify that p is identifiable.

d) Define consistent estimator of a parameter θ. State the Law of Large Numbers. Give a parametric statistical model and an example of a statistic that is consistent, and in one sentence explain if the Law of Large Numbers can be used to justify this property.

2. Consider the regular MLE case, with pdf $f(x;\theta)$, $\theta \in \Theta \subset R$ and $X \sim f$.

   a) Prove that
   $$\mathrm{E}_\theta \left(\frac{\partial \log f(X;\theta)}{\partial \theta} \right) = 0$$
   State any assumptions that are needed.

   b) Verify by direct calculation that this is true for the family of exponential distributions.

c) Explain whether or not this result applies to the Uniform(0,θ) family, where $\theta \in (0, \infty)$.

d) State the Cramér-Rao Lower Bound. Consider the Poisson family, with parameter λ being the population mean. In one or two sentences state how the Cramér-Rao lower bound is used to give some optimal property for the MLE.

3. Consider the family of geometric of the form that gives the number of failures *before* the first success. Let X denotes a r.v. with this density.

   a) Give this probability mass function.

   b) Suppose X_i, $i = 1, \ldots, n$ is a random sample of size n from this distribution. Give the likelihood and log likelihood function of θ.

   c) Obtain the Maximum Likelihood Estimator (MLE) $\hat{\theta}$ of θ using your choice of one of the functions in part (b).

d) Obtain the method of moments estimator. Is it different than the MLE in this case?

e) Calculate the information I(θ). Give the normal approximation to the distribution of $\hat{\theta}$.

f) Suppose that $n = 50$. For this data we observe $\sum_{i=1}^{n} x_i = 98$. Find the observed value of the MLE, that is the estimate, for this data. Give the (observed) approximate 95% confidence interval for θ based on the MLE.