CHAPTER 17

CONSTRUCTING TRANSFER FUNCTION-NOISE MODELS

17.1 INTRODUCTION

Hydrologists and other types of natural scientists often require a stochastic model which realistically describes the dynamic relationships connecting a single output series with one or more input series. For example, a stochastic model can be developed for formally modelling the mathematical linkage of a single output or response series such as seasonal riverflows to one or more input or covariate series such as precipitation and temperature. When inputs are incorporated into a stochastic model, the manner in which the input dynamically affects the output over time is mathematically modelled. For this reason, stochastic models which possess inputs are often referred to as *dynamic models*. Because the dynamic characteristics of the physical system being modelled are incorporated into the overall stochastic or dynamic model, this allows for more accurate forecasts to be made (see Chapter 18) and for more realistic values to be generated in simulation studies. Consequently, the main purpose of this chapter is to describe a flexible dynamic model which can be applied to many kinds of environmental problems where the data can be either nonseasonal or seasonal. The particular kind of dynamic model which is studied is called a *transfer function-noise model* where the acronym TFN is used for denoting transfer function-noise.

Throughout the chapter, *practical applications* are employed for clearly explaining how TFN models can be easily constructed by following the identification, estimation and diagnostic check stages of *model construction*. In particular, for model identification it is clearly pointed out how both a sound physical understanding of the problem being studied and comprehensive statistical procedures can be employed for designing an appropriate dynamic model to fit to a given set of time series. When designing a suitable dynamic model, usually it is most instructive and convenient to consider simpler models and to gradually increase the complexity of the model until a reasonable model is built. Accordingly, TFN models with a single input are entertained in the next section. The ways in which the residual CCF from Section 16.2.2, as well as two other techniques, can be used in model identification are thoroughly explained in Section 17.3 along with other model building techniques. For the application in Section 17.4.2 for a dynamic model having a single input, a TFN model is developed for relating average monthly upstream riverflows to downstream flows. Also, because the causality studies in Chapter 16 indicate that the average August temperature significantly affect the average annual flows of the Gota River, a dynamic model is constructed in Section 17.4.3 for rigorously describing this relationship.

For the situation where there is more than one input series, the efficacy of the model building techniques is clearly demonstrated in Section 17.5.4 by designing a TFN model to describe the dynamic relationships connecting a monthly river flow series in Canada to precipitation and temperature covariate series. In the process of selecting the most appropriate model to fit to the series, a number of useful modelling procedures are suggested. Often there are more than one precipitation and temperature series and a statistical procedure is presented for creating a single sequence to represent the precipitation or temperature series. This approach can be utilized in place of the rather ad hoc methods such as the Isohyetal and Thiessen polygon techniques (see Viessman et al. (1977) for a description of the Isohyetal and Thiessen polygon methods). To decide upon which series to include in the dynamic model and also design the form of the transfer function connecting a covariate series to the output, cross-correlation analyses of the residuals of models fitted to the series can be employed. The TFN model which is ultimately chosen by following contemporary modelling procedures can be used for applications such as forecasting and simulation and also providing insights into the physical characteristics of the phenomena being examined.

A TFN model is ideally designed for reflecting the physical characteristics of many natural systems. In a watershed, for example, the two basic equations that govern flow are the continuity equation or conservation of mass and the conservation of momentum. Delleur (1986) demonstrates that a TFN model can represent the discrete-time version of the continuous-time differential equation that is derived from the above two conservation principles. Because a TFN model can be readily calibrated by following the three stages of model construction given in this chapter and also the model reflects the physical characteristics of the system being described, the TFN model is ideally suited for real-world applications in hydrology and other sciences. For a discussion of the *physical justification* of ARMA models, the reader may wish to refer to Section 3.6.

Besides the TFN modelling applications presented in Sections 17.4.2, 17.4.3 and 17.5.4, other case studies are presented in Chapter 18 with the forecasting experiments. The intervention model, which in reality constitutes a special class of TFN models for modelling the affects of external interventions upon the mean level of the output series, can also handle multiple covariate series. Applications of intervention modelling for which there are input series are presented in Sections 19.5.4 and 22.4.2. Because TFN models work so well in practice, there are many published case studies of TFN in water resources, environmental engineering as well as other fields. Some of the many TFN modelling applications in the physical sciences include contributions in hydrology (Anselmo and Ubertini, 1979; Baracos et al., 1981; Hipel et al., 1992; Chow et al., 1983; Thompstone et al., 1983; Snorrason et al., 1984; Hipel et al., 1985; Maidment et al., 1985; Olason and Watt, 1986; Fay et al., 1987; Gurnell and Fenn, 1984; Lemke, 1990, 1991), dendroclimatology (Li, 1981, Ch. 8), modelling wastewater treatment plants (Capodaglio et al., 1992), and fish population studies (Noakes et al., 1987; Campbell et al., 1991; Welch and Noakes, 1991). Finally, a stochastic model closely related to the TFN model is defined and evaluated in Section 17.6.

17.2 TRANSFER FUNCTION-NOISE MODELS WITH A SINGLE INPUT

17.2.1 Introduction

For many natural systems it is known a priori if one variable, or set of variables, causes another. Lake levels, for instance, are obviously affected by precipitation. In situations where it is uncertain if one physical phenomenon causes another, the residual CCF discussed in Section 16.2.2 can be employed. For example, one may wish to find out if sunspots cause riverflows, as is done in Section 16.3.3. Whatever the case, subsequent to establishing the existence and direction of causality between two phenomena, a TFN model can be built to model mathematically the dynamical characteristics of the physical system.

Consider the situation where a variable X causes a variable Y. Let the set of observations for the variables X and Y be represented by the series X_t and Y_t, respectively, for $t = 0,1,2, \ldots, n$. The X_t and Y_t series may be transformed using a transformation such as the Box-Cox transformation in [3.4.30] to form the x_t and y_t series, respectively. When the data are seasonal, the seasonality in each series may be removed using the deseasonalization techniques presented in Section 13.2.2. No matter what kinds of transformations are performed before fitting the dynamic model, x_t and y_t will always be used to represent the transformed series.

Qualitatively, a TFN model can be written as

$$\text{output} = \text{dynamic component} + \text{noise}$$

The manner in which the input, x_t, dynamically affects the output, y_t, is modelled by the dynamic component. However, usually influences other than the input variable X will also affect Y. The accumulative effect of other such influences is called the noise or disturbance where the noise is usually correlated and may be modelled by an ARMA or ARIMA model. The dynamic and noise components are now discussed separately.

17.2.2 Dynamic Component

The dynamic relationship between X and Y can be modelled by a transfer function model as

$$\begin{aligned} y_t &= v_0 x_t + v_1 x_{t-1} + v_2 x_{t-2} + \cdots \\ &= v(B) x_t \end{aligned}$$

where $v(B) = v_0 + v_1 B + v_2 B^2 + \cdots$, is referred to as the transfer function and the coefficients, $v_0, v_1, v_2, \ldots,$ are called the *impulse response function* or *impulse response weights*. When there are nonzero means μ_y and μ_x for the y_t and x_t series, respectively, the transfer function model can be written in terms of deviations from the mean level as

$$y_t - \mu_y = v(B)(x_t - \mu_x)$$

The deterministic transfer function models how present and past values of X affect the current value of Y. The x_t series is often referred to as the *input, covariate* or *exogenous series*, while the y_t series is called the *output, response* or *endogenous series*.

Recall from [3.4.18], that for an ARMA model, the infinite MA operator is written in terms of two finite operators as

$$\Psi(B) = \frac{\theta(B)}{\phi(B)}$$

where $\Psi(B)$ is the infinite MA operator, $\theta(B)$ is the MA operator of order q and $\phi(B)$ is the AR operators of order p. In practical applications, only a small number of AR and MA parameters are required to model a given series. Hence, the infinite MA operator $\Psi(B)$ can be parsimoniously represented by $\frac{\theta(B)}{\phi(B)}$. In a similar fashion, the *transfer function*, $v(B)$, can be economically written as

$$\nu(B) = \frac{\omega(B)}{\delta(B)} = \frac{\omega_0 - \omega_1 B - \omega_2 B^2 - \cdots - \omega_m B^m}{1 - \delta_1 B - \delta_2 B^2 - \cdots - \delta_r B^r}$$

where $\omega(B)$ is the operator of order m in the numerator of the transfer function and $\omega_0, \omega_1, \cdots, \omega_m$, are the parameters of $\omega(B)$; and $\delta(B)$ is the operator of order r in the denominator of the transfer function and $\delta_1, \delta_2, \ldots, \delta_r$, are the parameters of $\delta(B)$.

If a system is *stable*, a finite incremental change in the input results in a finite incremental change in the output. In other words, a bounded change in the covariate variable causes a bounded change in the response. For the case of the transfer function model, this implies that the infinite series $\nu_0 + \nu_1 B + \nu_2 B^2 + \ldots$, converges for $|B| \leq 1$. Because the convergence of $\nu(B)$ is controlled by the operator $\delta(B)$ which is in the denominator of $\nu(B)$, the *requirement of stability* is that the roots of the characteristic equation $\delta(B) = 0$ lie outside the unit circle (Box and Jenkins, 1976, Ch. 10). Notice that the requirement of stability for a discrete transfer function model is analogous to the stationarity requirement for an ARMA model in Section 3.4.2 where the roots of $\phi(B) = 0$ must lie outside the unit circle.

In some situations, there may be a *delay time* before X affects Y. For instance, when an organic pollutant is discharged into a river, there may be a delay time before certain biological processes take place and the dissolved oxygen level of the river drops. If this delay time is denoted by b, where b is a positive integer for use in a model using evenly spaced discrete time points, the transfer function model can be written as

$$y_t - \mu_y = \nu(B)(x_t - \mu_x) = \frac{\omega(B)}{\delta(B)} B^b (x_t - \mu_x) \qquad [17.2.1]$$

When the parameters for the $\omega(B)$ and $\delta(B)$ operators are known, as is the case when they are estimated from the given data, the ν_k coefficients can be determined by equating coefficients of B^k in

$$\delta(B)\nu(B) = \omega(B)B^b$$

The ν_k weights can be more conveniently determined by expressing the above equation as

$$\delta(B)\nu_{k+b} = -\omega_k \quad \text{for } k = 1,2,\ldots \qquad [17.2.2]$$

where B operates on the subscript in the ν_{k+b} coefficient and therefore $B^j \nu_{k+b} = \nu_{k+b-j}$; $\nu_b = \omega_0$ and $\nu_k = 0$ for $k < b$; $\omega_k = 0$ for $k < 0$ and $k > m$.

As an example of how to use [17.2.2], consider the situation where $\nu(B) = \frac{\omega_0 B}{1-\delta_1 B}$ and it is necessary to determine the impulse response function where ω_0 and δ_1 are given. For this transfer function, [17.2.2] becomes

$$(1 - \delta_1 B)\nu_{k+1} = -\omega_k$$

Because there is a delay factor of one, $\nu_0 = 0$ and $\nu_1 = \omega_0$. When k=1 in the above equation

$$(1 - \delta_1 B)v_2 = -\omega_1 = 0$$

or

$$v_2 - \delta_1 v_1 = 0$$

Therefore,

$$v_2 = \delta_1 v_1 = \delta_1 \omega_0$$

For k=2

$$(1 - \delta_1 B)v_3 = -\omega_2 = 0$$

$$v_3 - \delta_1 v_2 = 0$$

$$v_3 = \delta_1 v_2 = \delta_1^2 \omega_0$$

In general,

$$v_k = \delta_1^{k-1} \omega_0$$

Because the v_k weights are calculated in a similar fashion to the ψ_k coefficients in the infinite MA operator for an ARMA model, the reader can refer to Section 3.4.3 for further examples of how to determine the impulse response weights.

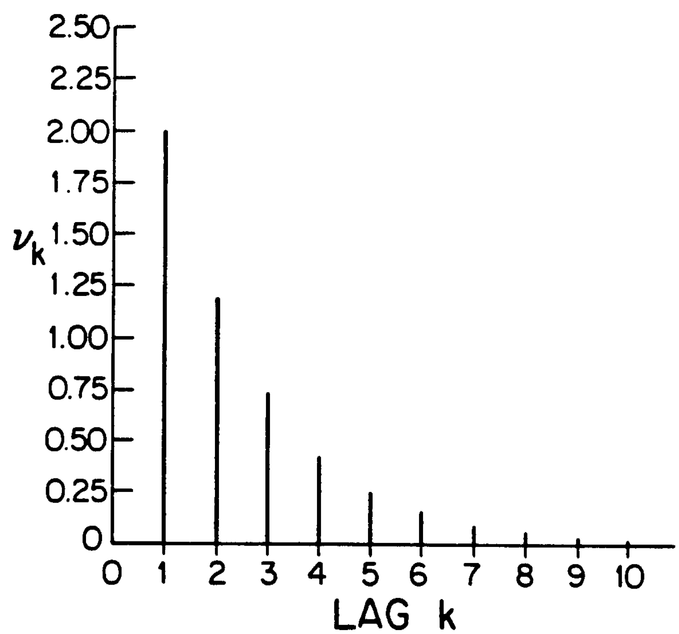

Figure 17.2.1. Impulse response function for $\omega_0 = 2$ and $\delta_1 = 0.6$.

When the calculated impulse response function is plotted for the case where $\omega_0 = 2$ and $\delta_1 = 0.6$, the impulse response function is as shown in Figure 17.2.1. Notice that $v_0 = 0$ due to the delay factor of one in the transfer function $v(B) = \frac{\omega_0 B}{(1-\delta_1 B)}$ and that the impulse response function attenuates from lag 2 onwards because of the operator $(1-\delta_1 B)$ in the denominator of the transfer function. The general form of the transfer function, especially the operator $\delta(B)$ in the denominator of the transfer function, allows for great flexibility in the design of a transfer function for ascertaining the effects of the input upon the output. For example, when a variable X affects Y after a delay factor of one and the magnitude of the effect upon y_t of each x_t decreases more and more for $t = t+1, t+2, t+3, \ldots$, then a transfer function of the form $v(B) = \frac{\omega_0 B}{(1-\delta_1 B)}$ may be appropriate.

If the input, x_t, in [17.2.1] is indefinitely held at some fixed value for a stable system, the output, y_t, will eventually reach an equilibrium point which is called the steady state level. Using the form of the transfer function model in [17.2.1], the steady state relationship is

$$y_t - \mu_y = g(x_t - \mu_x)$$

where g is called the steady state gain when x_t is held indefinitely at a fixed level. Suppose that $(x_t - \mu_x)$, which is the deviation of the input from the mean level, is kept at a value of +1. Then

$$\begin{aligned} y_t - \mu_y &= g1 \\ &= (v_0 + v_1 B + v_2 B^2 + \ldots)1 \\ &= v_0 + v_1 + v_2 + \cdots \end{aligned}$$

Consequently, for a stable system, the *steady state gain* is

$$g = \sum_{i=0}^{\infty} v_i = \frac{\omega(1)}{\delta(1)} \qquad [17.2.3]$$

For the case where

$$v(B) = \frac{\omega_0 B}{(1-\delta_1 B)} = \omega_0(1 + \delta_1 B + \delta_1^2 B^2 + \cdots)B$$

in [17.2.1], the steady state gain is simply

$$g = \frac{\omega_0}{1-\delta_1}$$

by substituting $B = 1$ into the equation.

17.2.3 Noise Term

In practice, a physical system cannot be realistically modelled by using only the deterministic transfer function model in [17.2.1]. As shown qualitatively below, usually there is noise left in the system after the deterministic dynamic effects of the input upon the output have been accounted for.

output − dynamic component = noise

Because the noise term, which is denoted by N_t, is often autocorrelated and not white, it can be conveniently modelled using the ARMA model in [3.4.4], as

$$\phi(B)N_t = \theta(B)a_t \qquad [17.2.4]$$

or

$$N_t = \frac{\theta(B)}{\phi(B)}a_t$$

where $\phi(B)$ and $\theta(B)$ are the AR and MA operators of orders p and q, respectively, and a_t is the white noise which is $IID(0,\sigma_a^2)$. Although the a_t series can be assumed to be $\text{IID}(0,\sigma_a^2)$ from a theoretical viewpoint, for practical reasons the a_t must be assumed to follow a given distribution in order to be able to obtain estimates for the model parameters. Consequently, the a_t are assumed to be $NID(0,\sigma_a^2)$ and, as demonstrated by practical applications, this assumption does not restrict the flexibility of the TFN model in any way. In fact, if the a_t series were not assumed to be normally distributed, it would be virtually impossible to obtain efficient estimates for the model parameters. As shown by the examples in this chapter and also Chapters 18, 19 and 22, for most environmental applications the noise term is stationary and therefore can be modelled using an ARMA model. Nevertheless, if the noise were nonstationary and differencing were required, it could be modelled using an ARIMA model.

17.2.4 Transfer Function-Noise Model

By combining [17.2.4] and [17.2.1], an overall TFN model for simultaneously modelling both the dynamic characteristics and noise contained in the system is formed as

$$\begin{aligned} y_t - \mu_y &= \nu(B)(x_t - \mu_x) + N_t \\ &= \frac{\omega(B)}{\delta(B)}B^b\,(x_t - \mu_x) + \frac{\theta(B)}{\phi(B)}\,a_t \end{aligned} \qquad [17.2.5]$$

Because the above model possesses both dynamic and stochastic or noise components, it could perhaps be referred to as a stochastic dynamic model. However, since all of the models discussed in this text are stochastic in nature, usually this TFN model is simply referred to as a dynamic model. Also, because the noise term models what the dynamic component cannot account for, it is assumed that N_t is independent of x_t in [17.2.5]. Since N_t is generated by a_t and x_t is generated by u_t in [16.2.3], this in turn means that the a_t series in [17.2.5] is independent of the u_t series in [16.2.3].

As noted by authors such as Abraham and Ledolter (1983, p. 338) and Vandaele (1983, pp. 263-264), the TFN model can be thought of as a generalization of the usual regression model for various reasons. Firstly, the model in [17.2.5] allows for the noise term to be correlated whereas the error term in a regression model is assumed to be white. Secondly, due to the flexible manner in which the transfer function in [17.2.5] is designed with an operator in both the denominator and numerator, the dynamic relationships between the output and input are more realistically modelled with a TFN model. As pointed out by Young (1984, p. 104), linear regression models are primarily utilized in the evaluation of static relationships among variables and are not generally suitable for use in dynamic systems analysis. Finally, as is also the case with regression analysis, one or more input series can be incorporated into a TFN model.

17.3 MODEL CONSTRUCTION FOR TRANSFER FUNCTION-NOISE MODELS WITH ONE INPUT

17.3.1 Model Identification

No matter what type of model is being fitted to a given data set it is recommended to follow the identification, estimation and diagnostic check stages of model development. As is the case for most of the models in this book, the three stages of model construction for a TFN model closely parallel those already described for models such as the ARMA model (see Chapters 5 to 7) and the seasonal models (Part VI). Nevertheless, some additional model building techniques are required, especially for model identification. When designing a TFN model, the number of parameters required in each of the operators contained in the dynamic and noise transfer functions in [17.2.5] must be identified. The three procedures for model identification described in this chapter are the empirical approach that has been used when modelling hydrological time series (Hipel et al., 1977, 1982, 1985), the technique of Haugh and Box (1977) which uses the residual CCF of Section 16.2.2, and the method of Box and Jenkins (1976) which is based upon suggestions by Bartlett (1935). The latter two methodologies rely heavily upon the results of cross-correlation studies and often the first procedure can be used in conjunction with either the second or third approaches. Subsequent to a description of the identification and other model building procedures, two practical applications are employed in Section 17.4 for demonstrating how they work in practice. These and other practical applications show that the empirical identification approach is usually the simplest to use in practice.

Empirical Identification Approach

The two major steps involved in model identification by the empirical method are as follows:

(i) Based upon an understanding of the physical phenomena that generated the time series and also the mathematical properties of the TFN model, identify the transfer function $v(B)$ in [17.2.5]. For example, when modelling a monthly time series where the input is a precipitation time series and the output is a riverflow sequence, it may be known from the physical characteristics of the watershed that the rainfall for the current month only causes direct runoff during the present time period and one month into the future. Therefore, it may be appropriate to employ in [17.2.5] a transfer function of the form

$$v(B) = \omega_0 - \omega_1 B = v_0 + v_1 B$$

When the present value of the covariate series causes an immediate change in the output, y_t, where this change attenuates as time progresses, a suitable transfer function may be

$$v(B) = \frac{\omega_0}{(1-\delta_1 B)} = \omega_0(1 + \delta_1 B + \delta_1^2 B^2 + \cdots)$$

$$= v_0 + v_1 B + v_1 B^2 + \cdots$$

The v_i coefficients decrease in value for increasing lag since $|\delta_1| < 1$ in order for the root of $(1-\delta_1 B) = 0$ to lie outside the unit circle. Hipel et al. (1977, 1982, 1985) present a number of practical applications where a physical understanding of the process in conjunction with some simple graphical identification techniques are utilized to design transfer functions for covariate series and also intervention series which are needed when the effects of external interventions upon the response series must be incorporated into the model (see Chapters 19 and 22).

(ii) After deciding upon the form of $v(B)$, identify the parameters needed in the noise term in [17.2.5]. To accomplish this, first fit the model in [17.2.5] to the series where it is assumed that the noise term is white and consequently the TFN model has the form

$$y_t - \mu_y = v(B)(x_t - \mu_x) + a_t$$

In practice, the noise term is usually correlated. Therefore, after obtaining the estimated residual series, $\hat{a}_t$, for the above model, the type of ARMA model to fit to the calculated noise series can be ascertained by following the usual procedures of model development for a single series described in Chapters 5 to 7. By using the identified form of N_t for the noise term along with the previously designed dynamic component, the TFN model in [17.2.5] is now completely specified.

Haugh and Box Identification Method

It seems logical that a model similar to the one in [17.2.5] that relates the x_t and y_t series, could be developed for connecting their innovation series given as μ_t and v_t, respectively. Recall from Section 16.2.2 that the μ_t disturbances are for the ARMA model fitted to the x_t series as

$$\phi_x(B)(x_t - \mu_x) = \theta_x(B) u_t$$

in [16.2.3]. Likewise, the v_t innovations are formed by prewhitening the y_t series using the ARMA model written as

$$\phi_y(B)(y_t - \mu_y) = \theta_y(B) v_t$$

in [16.2.4]. As shown by Haugh and Box (1977), a TFN model for the residual series can be written as

$$v_t = \nu'(B)u_t + N'_t \qquad [17.3.1]$$

where $\nu'(B) = \frac{\omega'(B)}{\delta'(B)}$ is the transfer function which has the same form as the transfer function in [17.2.5] except that a prime symbol is assigned to each parameter; $N'_t = \frac{\theta'(B)}{\phi'(B)} a_t$ is the ARMA noise term where the MA operator, $\theta'(B)$, and the AR operator, $\phi'(B)$, are designed in the same fashion as their counterparts in [3.4.4], [16.2.3] or [16.2.4]. In their paper, Haugh and Box (1977) derive the relationships between the transfer functions in [17.2.5] and [17.3.1] as

$$\nu(B) = \frac{\omega(B)}{\delta(B)} = \frac{\theta_y(B)}{\phi_y(B)} \frac{\omega'(B)}{\delta'(B)} \frac{\phi_x(B)}{\theta_x(B)} \qquad [17.3.2]$$

and

$$\frac{\theta(B)}{\phi(B)} = \frac{\theta_y(B)}{\phi_y(B)} \frac{\theta'(B)}{\phi'(B)} \qquad [17.3.3]$$

By knowing the form of the operators on the right hand sides of [17.3.2] and [17.3.3], the parameters needed for the model in [17.2.5] can be identified. The detailed steps required for executing this identification process are as follows, where the first two steps follow the development of the residual CCF described in Section 16.2.2:

(i) Determine the most appropriate ARMA models to fit to the x_t and also the y_t series by adhering to the three stages of model construction. In addition to the ARMA model parameters, estimates are also obtained for the innovation series in [16.2.3] and [16.2.4] at the estimation stage of model development. The estimated innovation or residual series, $\hat{u}_t$ and $\hat{v}_t$, are formed from the process of prewhitening the given x_t and y_t series in [16.2.3] and [16.2.4], respectively using the calibrated ARMA models.

(ii) By utilizing [16.2.6], calculate the residual CCF for the $\hat{u}_t$ and $\hat{v}_t$ series, along with the 95% confidence limits.

(iii) Based upon the characteristics of the residual CCF identify the parameters needed in the transfer function $\nu'(B)$ in [17.3.1]. As demonstrated by Haugh and Box (1977, p. 126), the theoretical CCF for u_t and v_t is directly proportional to the impulse response function given by $\nu'_0, \nu'_1, \nu'_2, \cdots$, here

$$\nu'(B) = \frac{\omega'(B)}{\delta'(B)} = \nu'_0 + \nu'_1 B + \nu'_2 B^2 + \cdots$$

is the transfer function in [17.3.1]. In particular, the kth parameter in $\nu'(B)$ is related to the theoretical CCF by the equation

$$\nu'_k = \sigma_u \sigma_v \rho_{uv}(k), \quad k = 0,1,2,\cdots \qquad [17.3.4]$$

where σ_u and σ_v are the theoretical standard deviations for μ_t and ν_t, respectively, while $\rho_{uv}(k)$ is the theoretical residual CCF in [16.2.5]. Therefore, the form of the transfer function can be identified from the residual CCF and when the quantities on the right hand side of [17.3.4] are replaced by their estimates in [16.2.6], an initial estimate can be obtained for ν'_k. If, for example, the residual CCF possesses values which are significantly different from zero only at lags 0, 1, and 2, the transfer function may be identified as

$$v'(B) = \omega'_0 - \omega'_1 B - \omega'_2 B^2 = v'_0 + v'_1 B + v'_2 B^2$$

When the value of the residual CCF at lag 0 is significantly different from zero, the values attenuate for increasing positive lags, and the residual CCF values at negative lags are not significant, then the impulse response function must be designed to mimic this behaviour. Accordingly, an appropriate transfer function may be

$$v'(B) = \frac{\omega'_0}{(1 - \delta'_1 B)} = \omega'_0(1 + \delta'_1 B + \delta'^2_1 B^2 + ...)$$

$$= v'_0 + v'_1 B + v'_2 B^2 + \cdots$$

The above transfer function is suitable because $|\delta'_1| < 1$ in order for the root of $(1 - \delta'_1 B) = 0$ to lie outside the unit circle and this in turn means that $(1 - \delta'_1 B)^{-1}$ causes the v'_i coefficients to decrease in absolute magnitude with increasing positive lags. A cyclic pattern in the residual CCF may indicate that $\delta'(B)$ should be at least second order in B.

(iv) The residual CCF from step (ii) can also be employed to ascertain the form of $\frac{\theta'(B)}{\phi'(B)}$ in [17.3.1]. As shown by Haugh and Box (1977), $\phi'(B) = \delta'(B)$ and $\theta'(B)$ should be at most of the order of $\delta'(B)$ or $\omega'(B)$.

(v) The results from steps (i) and (iii) can be substituted into [17.3.2] to obtain the form of $v(B)$. Likewise, the information from stages (i) and (iv) can be employed to get $\frac{\theta(B)}{\phi(B)}$ in [17.3.3]. The transfer function-noise model in [17.2.5] has now been completely identified.

Box and Jenkins Identification Procedure

By following the procedure of Box and Jenkins (1976) the model in [17.2.5] can be designed according to the following steps.

(i) Ascertain the most appropriate ARMA model in [16.2.3] to fit to the x_t series by utilizing the three stages of model construction presented in Part III. At the estimation stage, estimates are obtained for the ARMA model parameters and also the innovation series.

(ii) Using the ARMA filter, $\frac{\hat{\theta}_x(B)}{\hat{\phi}_x(B)}$, from step (i), transform the y_t series by employing

$$\hat{\beta}_t = \left[\frac{\hat{\theta}_x(B)}{\hat{\phi}_x(B)}\right]^{-1} y_t \qquad [17.3.5]$$

where the $\hat{\beta}_t$ sequence is usually not white noise since the filter in [16.2.4] is not used in [17.3.5].

(iii) After replacing the $\hat{v}_t$ series by the $\hat{\beta}_t$ sequence, use [16.2.6] to calculate the residual CCF for the $\hat{u}_t$ and $\hat{\beta}_t$ series.

(iv) Based upon the behaviour of the residual CCF from step (iii), identify the parameters required in the transfer function, $v(B)$, in [17.2.5]. As shown by Box and Jenkins (1976, p. 380), the theoretical CCF between the prewhitened input, u_t, and the correspondingly transformed output, β_t, is directly proportional to the impulse response function defined in Section 17.2.2. Consequently, the behaviour of the residual CCF can be utilized to identify the form of the transfer function in [17.2.5]. If, for instance, the residual CCF values are not significantly different from zero for negative lags, the estimated CCF at lag zero is significant, and the values attenuate for increasing positive lags, then the impulse response function must also follow this general behaviour. Therefore, an appropriate transfer function may be

$$v(B) = \frac{\omega_0}{1-\delta_1 B} = \omega_0(1+\delta_1 B+\delta_1 B^2+...)$$

$$= v_0 + v_1 B + v_2 B^2 + \cdots$$

since the v_i weights decrease in absolute magnitude with increasing positive lags due to the operator, $(1-\delta_1 B)$, with $|\delta_1| < 1$, in the denominator. A cyclic pattern in the residual CCF may imply that $\delta(B)$ should be at least second order in B. When the residual CCF possesses values which are significantly different from zero only at lags 0 and 1, the transfer function may be identified as

$$v(B) = \omega_0 - \omega_1 B = v_0 + v_1 B$$

(v) Subsequent to ascertaining the form of $v(B)$, determine the parameters needed in the noise term in [17.2.5]. Upon obtaining moment estimates for the parameters in $v(B)$, calculate the noise series from [17.2.5] by using

$$\hat{N}_t = (y_t - \bar{y}) - \hat{v}(B)(x_t - \bar{x})$$

where $\bar{y}$ and $\bar{x}$ are the sample means for μ_y and μ_x, respectively. By examining graphs such as the sample ACF and the sample PACF of $\hat{N}_t$, identify the ARMA model needed to fit to the noise series (see Section 5.3). Box and Jenkins (1976, pp. 384-385) also give a second procedure for identifying N_t where the sample CCF for $\hat{u}_t$ and $\hat{\beta}_t$, must first be calculated. The entire TFN model has now been tentatively designed.

Comparison of Identification Methods

The foregoing three identification procedures possess different inherent assets and liabilities. Although the empirical approach is straightforward and simple to apply, experience and understanding are required in order to properly identify the parameters needed in $v(B)$ at step (i). Because the empirical approach does not consider cross-correlation information in the first step, either the method of Haugh and Box or else Box and Jenkins could be utilized to check that $v(B)$ is properly designed. An advantage of the Haugh and Box method is that the residual CCF results that are employed for detecting causal relationships in Section 16.2.2 are also used for model identification. However, due to the relationships given in [17.3.2] and [17.3.3], the procedure is rather complicated and care should be taken that the model is not over-specified by having too many parameters. If this problem is not found at the identification stage, it may be detected at the estimation stage where some of the parameters may not be significantly different from zero. Common factors that appear in both the numerator and denominator of a transfer function should, of course, be left out of the model.

When using the Box and Jenkins approach, the form of $v(B)$ is identified directly from the CCF for the $\hat{u}_t$ and $\hat{\beta}_t$ series. However, in most cases $\hat{\beta}_t$ will not be white noise and therefore the estimated values of the CCF are correlated with one another (Bartlett, 1935). Consequently, caution should be exercised when examining the estimated CCF for $\hat{u}_t$ and $\hat{\beta}_t$.

In addition to the three identification methods presented in this section, other techniques are also available. For instance, Liu and Hanssens (1982) propose a procedure for identifying the transfer function parameters needed in $v(B)$ in [17.2.1] based upon least squares estimates of the transfer function weights using the original or filtered series. The corner method of Beguin et al. (1980) is then used to identify the order of the $\omega(B)$ and $\delta(B)$ operators. An advantage of the technique of Liu and Hanssens (1982) is that it is specifically designed for handling the situation where there are multiple input series to the TFN model as in [17.5.3]. A drawback of their technique is that it is fairly complicated to use in practice and, therefore, is not as convenient to employ as the empirical approach.

All of the identification techniques discussed in this chapter are designed for use in the time domain. An alternative approach to transfer function identification in the time domain is to identify a transfer function using frequency domain or spectral methods such as those suggested by Box and Jenkins (1976) and Priestley (1971). However, as noted by Liu and Hanssens (1982), spectral techniques are difficult to apply to practical problems.

When designing a TFN model, it is not necessary to adhere strictly to a given identification procedure. In certain situations, it may be advantageous to combine various steps from two or three of the three aforementioned identification methods which were discussed in detail. For example, either the method of Haugh and Box (1977) or the technique of Box and Jenkins (1976) could be employed to identify $v(B)$ in [17.2.5]. Step (ii) in the empirical approach could then be utilized to determine the form of N_t. In general, no matter what identification method is being utilized it is advantageous to begin with a fairly simple model, since the presence of too many parameters may cause the estimation procedure to become unstable. Because the results of the identification procedure can often be rather ambiguous, usually two or three possible models are suggested. If a suitable model is not included within the set of identified models, this will be detected at the estimation or diagnostic check stages of model construction. Either a more complicated model will be needed or further simplification will be possible due to having too many parameters.

17.3.2 Parameter Estimation

Following the identification of one or more plausible TFN models, efficient estimates must be simultaneously obtained for all of the model parameters along with their standard errors (SE's) of estimation. Because the a_t's are assumed to be normally, independently distributed, MLE's can be conveniently calculated for the model parameters along with their SE's. Appendix A17.1 explains how MLE's can be determined for the TFN model in [17.2.5]. As explained in that appendix, because the noise term in the TFN follows an ARMA process, one can expand an estimator developed for ARMA models for use in obtaining MLE's of the parameters in a TFN model. Moreover, an estimation procedure developed for use with a TFN model can also be employed with the intervention model of Chapters 19 and 22.

Often more than one tentative TFN model are initially identified. Subsequent to estimating the model parameters separately for each model, automatic selection criteria such as the AIC in [6.3.1] and the BIC in [6.3.5] can be utilized to assist in selecting the most appropriate model. Figure 6.3.1 outlines how the AIC or another appropriate automatic selection criterion can be incorporated into the three stages of model selection. If a suitable range of models is considered, it has been found in practice that the model possessing the minimum AIC value usually satisfies diagnostic tests of the model residuals. Nevertheless, the model or set of models that are thought to be most suitable should be thoroughly checked in order to ascertain if any further model improvements can be made.

17.3.3 Diagnostic Checking

The innovation sequence, a_t , is assumed to be independently distributed and a recommended procedure for checking the whiteness assumption is to examine a plot of the RACF (residual autocorrelation function) along with confidence limits. The RACF, $r_{\hat{a}\hat{a}}(k)$, can be calculated by replacing both $\hat{u}_t$ and $\hat{v}_t$ by $\hat{a}_t$ in [16.2.6] or else using [7.3.1]. Since $r_{\hat{a}\hat{a}}(k)$ is symmetric about lag zero, the RACF is only plotted against lags for $k = 1$ to $k \approx n/4$, along with the 95% confidence limits explained in Section 7.3.2. Although a plot of the RACF is the best whiteness test to use, other tests which can be employed include the cumulative periodogram in [2.6.2] and the modified Portmanteau test.

Three versions of the Portmanteau test for whiteness of the $\hat{a}_t$ residuals are given in Section 7.3.3 in [7.3.4] to [7.3.6]. In particular, the Portmanteau test in [7.3.6] is written as

$$Q_L = n\sum_{k=1}^{L} r_{\hat{a}\hat{a}}^2(k) + \frac{L(L+1)}{2n} \qquad [17.3.6]$$

where n is the number of data, $r_{\hat{a}\hat{a}}(k)$ is the residual CCF from [16.2.6] (replace both the $\hat{u}_t$ and $\hat{v}_t$ series by the $\hat{a}_t$ series in [16.2.6]), and L is a suitably chosen lag such that after L time periods a_t and a_{t-L} would not be expected to be correlated. For instance, when deseasonalized monthly data are being used in a TFN model, L should be chosen at least as large as lag 12 to make sure that there is no correlation between residuals which are separated by one year. Because Q_L is distributed as $\chi^2(L-p-q)$, where p and q are the orders of the AR and MA operators, respectively in the ARMA model for N_t , significance testing can be done to see if significant correlation of the model residuals is present.

If the residuals are correlated, this suggests some type of model inadequacy is present in the noise term or the transfer function, or both of these components. To ascertain the source of the error in the model, the CCF for the $\hat{u}_t$ and $\hat{a}_t$ sequences can be studied (leave $\hat{u}_t$ as $\hat{u}_t$ and replace $\hat{v}_t$ by $\hat{a}_t$ in [16.2.6] to estimate $r_{\hat{u}\hat{a}}(k)$). Because the u_t and a_t series are assumed to be independent of one another, the estimated values of $r_{\hat{u}\hat{a}}(k)$ should not be significantly different from zero where one standard error is approximately $n^{-1/2}$ when the CCF is normally distributed. When a plot of $r_{\hat{u}\hat{a}}(k)$ from $k \approx -n/4$ to $k \approx n/4$ along with chosen confidence limits indicate whiteness while significant correlations are present in $r_{\hat{a}\hat{a}}(k)$, the model inadequacy is probably in the noise term, $\hat{N}_t$. The form of the RACF for the $\hat{a}_t$ series could suggest appropriate modifications to the noise structure. However, if both $r_{\hat{a}\hat{a}}(k)$ and $r_{\hat{u}\hat{a}}(k)$ possess one or more significant values, where $r_{\hat{u}\hat{a}}(k)$ only has large values at non-negative lags, this could mean that the transfer function for the input series is incorrect and the noise term may or may not be suitable. When feedback is indicated by significant values of $r_{\hat{u}\hat{a}}(k)$ at negative lags, a multivariate model (see Part VII) should be considered rather than a TFN model.

Even though it is probably most informative to examine a plot of $r_{\hat{u}\hat{a}}(k)$ along with the 95% confidence limits, modified Portmanteau tests can also be employed to check if there are problems with the TFN model. To see whether or not $r_{\hat{u}\hat{a}}(k)$ has significantly large values at non-negative lags, the following modified Portmanteau statistic can be calculated.

$$Q_L = n^2 \sum_{k=0}^{L} r_{\hat{u}\hat{a}}^2(k)/(n-k) \qquad [17.3.7]$$

where Q_L is distributed as $\chi^2(L-r-m)$, and r and m are the orders of the $\delta(B)$ and $\omega(B)$ operators, respectively, in the transfer function for χ_t in [17.2.5]. If the calculated Q_L statistic is greater than the value of $\chi^2(L-r-m)$ from the tables at the chosen significance level, this could mean that the transfer function is incorrect and the noise term may or may not be suitable. By choosing more appropriate values of r and m, a model which passes this test can usually be found.

To check if $r_{\hat{u}\hat{a}}(k)$ has significantly large values at negative lags, the modified Portmanteau statistic can be determined using

$$Q_L = n^2 \sum_{k=-1}^{-L} r_{\hat{u}\hat{a}}^2(k)/(n+k) \qquad [17.3.8]$$

where Q_L is distributed as $\chi^2(L)$. If significance testing indicates that there are values of $r_{\hat{u}\hat{a}}(k)$ which are significantly different from zero at negative lags, this implies feedback and a multivariate model should be used (see Part IX) instead of a TFN model. Because $r_{\hat{u}\hat{a}}(-k) = r_{\hat{a}\hat{u}}(k)$, equation [17.3.8] can be equivalently written as

$$Q_L = n^2 \sum_{k=1}^{L} r_{\hat{a}\hat{u}}^2(k)/(n-k) \qquad [17.3.9]$$

Besides being independently distributed, the a_t sequence is assumed to follow a normal distribution and possess a constant variance (homoscedasticity). In Sections 7.4 and 7.5, tests are presented for checking the normality and homoscedastic suppositions, respectively. As noted in Section 3.4.5 as well as other parts of the book, in practice it has been found that a suitable Box-Cox transformation of the Y_t and/or X_t series can often correct non-normality and heteroscedasticity in the residuals.

Whenever problems arise in the model building process, suitable model modifications can be made from information at the diagnostic check and identification stages. Subsequent to estimating the model parameters for the new model, the modelling assumptions should be checked to see if further changes are necessary.

17.4 HYDROLOGICAL APPLICATIONS OF TRANSFER FUNCTION-NOISE MODELS WITH A SINGLE INPUT

17.4.1 Introduction

Two hydrological applications are presented for clearly explaining how TFN models are constructed in practice. In the first application, the residual CCF described in Section 16.2.2, is employed for determining the statistical relationship between monthly flows in the tributary of a river and the flows downstream in the main river. By using each of the three identification methods described in Section 17.3.1, tentative TFN models are designed for modelling the two monthly riverflow series and the most appropriate model is verified by diagnostic checking.

In the second application, a TFN model is designed for formally modelling one of the causal relationships discovered using the residual CCF in Section 16.2.3. The residual CCF studies from Chapter 16 can be considered as part of the *exploratory data analysis* stage where simple graphical and statistical tools are employed for detecting important statistical characteristics of the data (Tukey, 1977). At the *confirmatory data analysis* step, the TFN model in [17.2.5] can be utilized to formally model and confirm the mathematical relationships which are discovered at the exploratory data analysis stage. Accordingly, a dynamic model is developed for formally describing the connections between the annual Gota River flows and monthly temperatures.

17.4.2 Dynamic Model for the Average Monthly Flows of the Red Deer and South Saskatchewan Rivers

Identification

The South Saskatchewan (abbreviated as S.Sask.) River originates in the Rocky Mountains and flows eastward on the Canadian Prairies across the province of Alberta to Saskatchewan, where it joins the North Saskatchewan River northwest of the city of Saskatoon. These two rivers form the Saskatchewan River which flows into Lake Winnipeg in Manitoba, which in turn drains via the Nelson River into Hudson Bay. A major tributary of the S.Sask. River is the Red Deer River which connects to the S.Sask. River near the Alberta-Saskatchewan border. Average monthly flows in m^3/s are available from Environment Canada (1979a,b) for the Red Deer River near the city of Red Deer, Alberta and also for the S.Sask. River near Saskatoon, Saskatchewan. Saskatoon is located approximately 800 km downstream from the city of Red Deer and the area of the basin drained by the S.Sask. River at Saskatoon is 139,600 km^2, whereas an area of 11,450 km^2i, is drained by the Red Deer River at Red Deer.

Because the Red Deer River flows into the S.Sask. River it is obvious that the Red Deer River contributes to the overall flow of the S.Sask. River. However, even though the direction of causality can be easily physically justified a priori without a cross-correlation study, the results from a cross-correlation analysis can be employed to validate statistically the known causal relationship and also to design a TFN model that mathematically describes the dynamic connection between the input flows from the Red Deer River and output flows in the S.Sask. River.

Before obtaining the residual CCF, the flows must be prewhitened. When prewhitening a series it may be necessary to transform the data using the Box-Cox transformation given in [3.4.30]. Previously, hydrologists found by experience that a natural logarithmic transformation (i.e. $\lambda = 0$ in [3.4.30] and c = 0 when there are no zero flows) can preclude problems with heteroscedasticity and non-normality in the model residuals. Accordingly, for the time period from January, 1941, until December, 1962, the 264 values of the monthly flows for both the Red Deer and S.Sask. Rivers are transformed using natural logarithms. Subsequent to this, by using the deseasonalization procedure in [13.2.3], each time series is deseasonalized by subtracting out the monthly mean and dividing this by the monthly standard deviation for each data point in the logarithmic transformed data. The year 1962 is selected as the final year for which data are used because a large dam came into operation on the S.Sask. River after that time. By following the three stages of model construction given in Chapters 5 to 7, the most appropriate models from [16.2.3] and [16.2.4] to fit to the x_t and y_t series, respectively, are found to be ARMA(1,1) models. In Table 17.4.1, the MLE's and corresponding SE's for the model parameters are presented where the x_t series refers to the Red Deer flows and the y_t sequence represents the S.Sask. flows after taking natural logarithms and deseasonalizing the data.

Table 17.4.1. Parameter estimates for the Red Deer River and S.Sask. River ARMA(1,1) models.

RIVERS	PARAMETERS	MLE'S	SE'S
	$\phi_{x,1}$	0.845	0.045
Red Deer	$\theta_{x,1}$	0.292	0.080
x_t	σ_u^2	0.482	
	$\phi_{y,1}$	0.819	0.050
S.Sask.	$\theta_{y,1}$	0.253	0.084
y_t	σ_v^2	0.507	

The estimated white noise series, $\hat{u}_t$ and $\hat{v}_t$, for the x_t and y_t series, respectively, are automatically calculated at the estimation stage. By utilizing [16.2.6], the residual CCF in Figure 17.4.1 is calculated along with approximate 95% confidence limits. The large values at lags 0 and 1 statistically confirm the known physical fact that the Red Deer River causes flows in the S.Sask. River and not vice versa. As outlined in Table 16.2.1, because the residual CCF contains values which are only significantly different from zero at non-negative lags, there is unidirectional causality from X to Y. In Figure 17.4.1, the value of the residual CCF at lag -4 which just crosses the 95% confidence limits, can be attributed to chance.

As demonstrated by Figure 17.4.2, when the CCF for the x_t and y_t series are plotted the kind of causality cannot be statistically ascertained (note that the approximate 95% confidence limits in Figure 17.4.2 are derived for independent series). The large values at negative, zero and positive lags hide the known reality that the Red Deer River is a tributary of the S.Sask. River. Consequently, practitioners are urged to examine cautiously any CCF study where proper statistical procedures have not been followed.

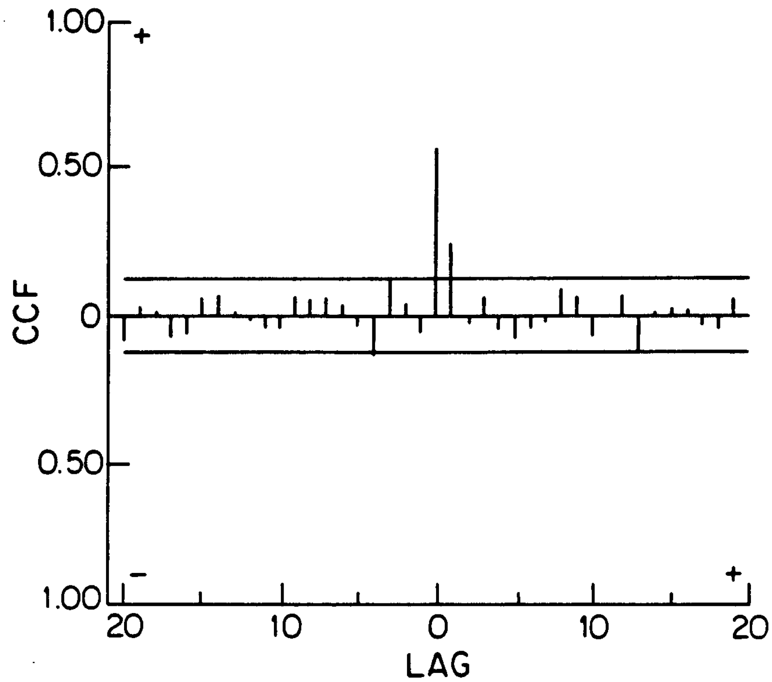

Figure 17.4.1. Residual CCF for the Red Deer and S.Sask. riverflows.

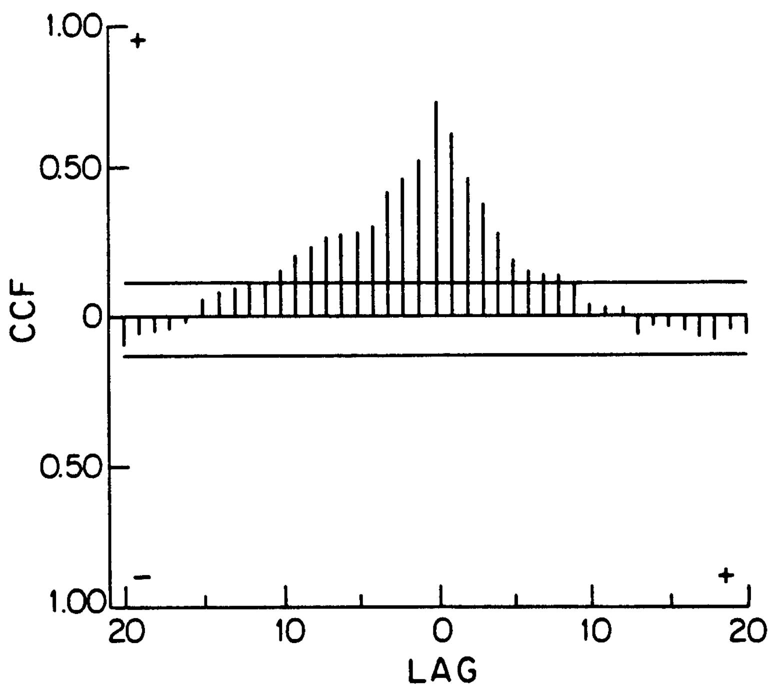

Figure 17.4.2. CCF for the deseasonalized logarithmic flows of the Red Deer and S.Sask. Rivers.

To explain how to design a TFN model for the x_t and y_t series for the Red Deer and S.Sask. Rivers, respectively, each of the three identification methods described in Section 17.3.1 is explained separately.

Empirical Identification Approach: Because monthly flows are being considered and also Saskatoon is about 800 km downstream from the city of Red Deer, from a physical point of view it would be expected that current Red Deer riverflows would affect the riverflows at Saskatoon during the present month and perhaps one month into the future. Consequently, a suitable transfer function may be

$$v(B) = \omega_0 - \omega_1 B$$

Assuming that the noise term in [17.2.5] is white noise, a TFN model is fitted to the x_t and y_t sequences, where the entries of the noise series are estimated along with the other model parameters. Using the standard model building procedures presented in Chapters 5 to 7, this series is found to be best described by an ARMA (1,1) model.

Haugh and Box Identification Method: Due to the large values at lags 0 and 1 of the residual CCF in Figure 17.4.1, the transfer function $v'(B)$ in [17.3.1] is identified to be

$$v'(B) = \omega'_0 - \omega'_1(B)$$

By employing [17.3.2] and also the results in Table 17.4.1, the transfer function in [17.2.5] which links x_t and y_t is calculated to be

$$v(B) = \frac{(1-0.253B)}{(1-0.819B)}(\omega'_0 - \omega'_1 B)\frac{(1-0.845B)}{(1-0.292B)}$$

$$\approx \omega'_0 - \omega'_1 B = \omega_0 - \omega_1 B$$

where the AR and MA operators can be dropped because $\theta_y(B) \approx \theta_x(B)$ and $\phi_y(B) \approx \phi_x(B)$ when the relative magnitudes of the standard errors are considered. Note that if it had been advantageous to get moment estimates for ω'_0 and ω'_1, [17.3.4] could have been utilized.

Since $\phi'(B) = \delta'(B)$, the order of the operator $\phi'(B)$ is zero. The order of $\theta'(B)$ should be at most of the order of $\delta'(B)$ or $\omega'(B)$ and, therefore, should be zero or one. Consequently, from [17.3.3] the noise term in [17.2.5] should be either ARMA (1,1) or else ARMA (1,2).

Box and Jenkins Procedure: By using [17.3.5], the $\hat{\beta}_t$ sequence is determined. The residual CCF for the $\hat{u}_t$ and $\hat{\beta}_t$ series is very similar to the plot in Figure 17.4.1 where there are large values only at lags 0 and 1. Accordingly, an appropriate transfer function for use in [17.2.5] is

$$v(B) = \omega_0 - \omega_1 B$$

Employing the same procedure used with the empirical identification method, the noise term is identified to be ARMA (1,1).

Parameter Estimation

The MLE's for the dynamic model linking x_t and y_t are listed in Table 17.4.2 where it is assumed that $\omega(B)$ is first order, $\delta(B)$ is of order zero, and the noise term is ARMA (1,1). The difference equation form of this model is written as

$$y_t = (0.572 + 0.238B)x_t + \frac{(1 - 0.494B)}{(1 - 0.856B)} a_t \qquad [17.4.1]$$

When the noise term is considered to be ARMA (1,2) the second MA parameter is not significantly different from zero and the value of the AIC is increased. Consequently, the simpler model in [17.4.1] is justified.

Table 17.4.2. Parameter estimates for the Red Deer-S.Sask. TFN model.

Parameters	MLE's	SE's
ω_0	0.572	0.049
ω_1	-0.238	0.049
ϕ_1	0.856	0.051
θ_1	0.494	0.085
σ_a^2	0.310	

Diagnostic Checking

The model in [17.4.1] satisfies the main modelling assumptions. In particular, the plot in Figure 17.4.3 of the RACF for the estimated a_t sequence along with 95% confidence limits reveals that the $\hat{a}_t$ series is white noise. The sample CCF and 95% confidence limits for the $\hat{u}_t$ and $\hat{a}_t$ sequences are displayed in Figure 17.4.4. Because the estimated values of the CCF and also the RACF in Figure 17.4.3 are not significantly different from zero, the transfer function and noise term are properly designed. Other diagnostic checks indicate that the $\hat{a}_t$ sequence is homoscedastic (see Section 7.5) and approximately normally distributed (see Section 7.4). Furthermore, the residual variance of 0.507 for the S.Sask. model in Table 17.4.1 is reduced by 39% to a value of 0.310 for the dynamic model in Table 17.4.2.

Concluding Remarks

Besides describing the dynamic relationship between the Red Deer and S.Sask. River, the model in [17.4.1] can be employed for applications such as forecasting and simulation. In fact, because the TFN model in Table 17.4.2 has a smaller residual variance than the ARMA (1,1) model in Table 17.4.1 for the S.Sask. River, it should produce more accurate forecasts. Forecasting with TFN models is explained and illustrated in Chapter 18.

17.4.3 Dynamic Model for the August Temperatures and Annual Flows of the Gota River

As presented in Table 16.3.3 for the causality studies of Section 16.3.3, the monthly temperature series are correlated with various annual riverflow series and the Beveridge wheat price index. The causality relationships which are found using the residual CCF as an exploratory data analysis tool can be further substantiated by developing a TFN model as a confirmatory data analysis tool. For illustrative purposes, one of the causal relationships in Table 16.3.3 is formally modelled here using a TFN model.

From Table 16.3.3, it can be seen that there are significant values of the residual CCF at lag zero between the flows of the Gota River and each of four monthly temperature series. This relationship is displayed graphically in Figure 16.3.3 for the case of the residual CCF for the August temperatures and the Gota riverflows. By following the model construction phases outlined in Section 17.3, an appropriate dynamic model can be developed (Hipel et al., 1985). When all four temperature series are used as covariate series in a TFN model, the transfer function parameter estimates for June, July and September are not significantly different from zero and can therefore be left out of the model. Whereas the residual CCF can only be used for pairwise comparisons, the TFN model can be employed to ascertain the most meaningful relationship when there are multiple covariate series and a single response or output series. For the case where only the August temperatures are used as a covariate series, the parameter estimates and SE's errors are listed in Table 17.4.3 while the difference equation for the model which follows the format of [17.2.5] is written as

$$Y_t - 535.464 = -23.869(X_t - 15.451) + (1 - 0.672B + 0.329B^2)^{-1}a_t \qquad [17.4.2]$$

where Y_t represents the annual flows of the Gota River and X_t stands for the monthly temperatures. Besides portraying the dynamic relationship between the Gota River flows and August temperatures, the model in [17.4.2] can, of course, be employed for forecasting and simulation.

17.5 TRANSFER FUNCTION-NOISE MODELS WITH MULTIPLE INPUTS

17.5.1 Introduction

In many situations, more than one input series is available for use in a TFN model and by incorporating all the relevant covariate series into the TFN model a dynamic model can be developed for producing more accurate forecasts and more realistic simulated values. For example, for the hydrometeorological application presented in Section 17.5.4, a flexible TFN model is constructed, where the average monthly precipitation and temperature series constitute the covariate series which affect the output or response consisting of average monthly riverflows. As demonstrated by this application, an inherent advantage of TFN modelling is that a TFN model with multiple inputs can be designed almost as easily as a model with a single input. In fact, the model building tools of Section 17.3 can easily be extended for use with a TFN model having multiple inputs.

In addition to using the most comprehensive statistical tools for use in model construction, the practitioner should exercise a lot of common sense and good judgement. As is the case for the other kinds of models considered in this text, TFN model building is in essence both an art and a science. The *art* of model building comes into play when the modeller uses his or her knowledge about the physical aspects of the problem to decide upon which covariate series should be incorporated into the TFN model and the general manner in which this should be done. For instance, for the application in Section 17.5.4, a suitable TFN model is developed by first considering simpler models which provide guidance as to how a more complex TFN model can be constructed. In the process of doing this, a simple procedure is suggested for creating a single input series which more than one precipitation or temperature series are available. By employing appropriate statistical and stochastic methods, the efficacy of the decisions made in the art of

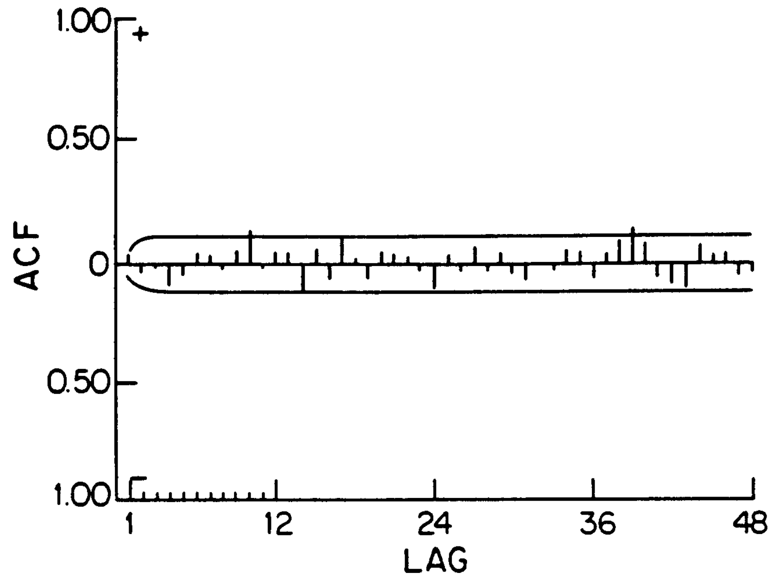

Figure 17.4.3. RACF for the Red Deer - S.Sask. TFN model.

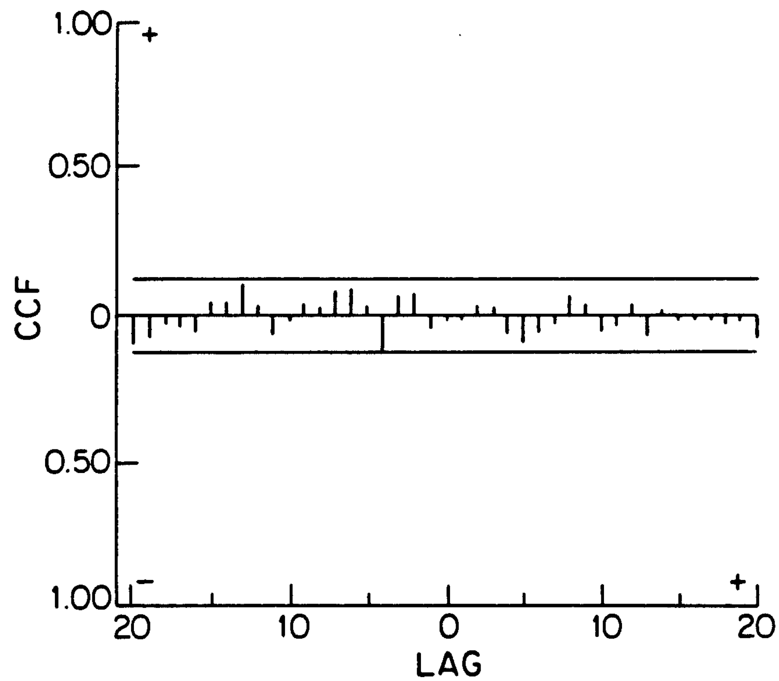

Figure 17.4.4. CCF of $\hat{u}_t$ and $\hat{a}_t$ for the Red Deer ARMA model and the Red Deer S.Sask. TFN model, respectively.

Table 17.4.3. Parameter estimates for the August temperature - Gota River flow transfer function-noise model.

PARAMETERS	MLE'S	SE'S
ω_0	-23.869	4.285
ϕ_1	0.672	0.077
ϕ_2	-0.329	0.077
σ_a	5.71×10^3	-

model building can be rigorously checked using the *science* of model construction. Consequently, there is interactive feedback when using both art and science for building TFN models, or for that matter, any other type of stochastic model.

17.5.2 Model Description

Qualitatively, the TFN model can be written as

output = dynamic component + noise

The dynamic component consists of a transfer function for each covariate series which describes how each input dynamically affects the output. As is also the case for a TFN model with one input in [17.2.5], the autocorrelated noise can be modelled using an ARMA model.

More precisely, a TFN model with multiple inputs can be written in the form

$$(y_t - \mu_y) = f(\mathbf{k},\mathbf{x},t) + N_t \qquad [17.5.1]$$

where t is discrete time, y_t is the output or response variable, μ_y is the mean of the y_t series, N_t is the stochastic noise term which may be autocorrelated, and $f(\mathbf{k},\mathbf{x},t)$ is the dynamic component of y_t . The dynamic component includes a set of parameters $\mathbf{k}$ and a group of covariate series $\mathbf{x}$. When required, both the response variable and one or more of the input variables may be transformed using a suitable Box-Cox transformation from [3.4.30]. As noted earlier, the reasons for transforming the series include stabilizing the variance and improving the normality assumption of the white noise series which is included in N_t . It should be pointed out that the same Box-Cox transformation need not be applied to all of the series. If the series are seasonal, subsequent to invoking appropriate Box-Cox transformations each series can be deseasonalized separately by employing the procedures of Section 13.2.2. Following this, identification procedures can be utilized to see which parameters should be included in the model in [17.5.1].

Included in the dynamic component of the model are the effects of all the input series upon the output. In general, if there are I input covariate series the dynamic component of the model is given by

$$f(\mathbf{k},\mathbf{x},t) = \sum_{i=1}^{I} v_i(B)(x_{ti} - \mu_{xi}) \qquad [17.5.2]$$

where x_{ti} is the ith input series which may be suitably transformed and μ_{xi} is the mean of x_{ti} . The ith transfer function which reflects the manner in which the ith input series, x_{ti} , affects y_t , is written as

$$v_i(B) = \frac{\omega_i(B)}{\delta_i(B)} B^{b_i}$$

$$= \frac{(\omega_{0i} - \omega_{1i}B - \omega_{2i}B^2 - \cdots - \omega_{m_i i}B^{m_i})B^{b_i}}{(1 - \delta_{1i}B - \delta_{2i}B^2 - \cdots - \delta_{r_i}B^{r_i})}$$

where m_i and r_i are the orders of the operators $\omega_i(B)$ and $\delta_i(B)$, respectively, and b_i is the delay time required before x_{ti} affects y_t. Notice that the ith transfer function in [17.5.2] is identical to the one utilized in [17.2.1] and [17.2.5], except that the subscript i has been added to indicate that $v_i(B)$ is the transfer function for the ith input series x_{ti}.

In practice, usually only a few parameters are required in each transfer function and therefore m_i and r_i are 0 or 1 (see the applications in this chapter as well as Chapters 18, 19 and 22). Given the parameters for the $\omega_i(B)$ and $\delta_i(B)$ operators, it may be required to estimate the v_{ji}, $j = 0,1,2,\ldots,$ coefficients in the operator

$$v_i(B) = (v_{0i} + v_{1i}B + v_{2i}B^2 + \cdots)$$

$$= \frac{\omega_i(B)B^{b_i}}{\delta_i(B)}$$

These coefficients can be calculated in exactly the same manner as they are in Section 17.2.2 for the parameters of $v(B)$ which is the transfer function used for a TFN model with one input.

The noise component of the TFN model having multiple inputs is defined by

$$N_t = y_t - f(\mathbf{k},\mathbf{x},t)$$

That is, the noise term of the model is simply the difference between the response variable, y_t, and the dynamic component. The form of the noise term, N_t, is not restricted to any particular form, but usually it is assumed to be an ARMA process as in [17.2.4]. Furthermore, as noted in Section 17.2.3, the white noise component of the ARMA model is usually assumed to be $NID(0,\sigma_a^2)$. Finally, because the noise term models the portion of y_t which is not explained by the dynamic component, N_t is independent of each x_{ti} series. Equivalently, since the a_t disturbances drive N_t and each x_{ti} series can be thought of as being generated by its own residual series, the a_t sequence is independent of the white noise series for a given input series which can be formed by prewhitening the x_{ti} sequence (see Section 16.2.2 for a discussion of prewhitening). Finally, the x_{ti} series or their residuals formed by prewhitening are not assumed to be independent of one another in a TFN model.

In summary, by combining the dynamic and noise components, the overall TFN model with I inputs can be written as

$$(y_t - \mu_y) = f(\mathbf{k},\mathbf{x},t) + N_t \qquad [17.5.3]$$

$$= \sum_{i=1}^{I} \frac{\omega_i(B)}{\delta_i(B)} B^{b_i}(x_{ti} - \mu_{xi}) + \frac{\theta(B)}{\phi(B)} a_t$$

17.5.3 Model Construction

When developing a TFN model with multiple inputs, the parameters required in each transfer function within the overall dynamic component plus the orders of the operators in the noise term, must be identified. Subsequent to this, MLE's can be obtained for the model parameters and the validity of the model verified by invoking appropriate diagnostic checks. Because the TFN model with multiple inputs in [17.5.3] is a straightforward extension of the single input model in [17.2.5], most of the construction tools presented in Section 17.3 for a TFN having one input can be used for the multiple input case. The purpose of this section is to clearly point out what special problems can arise when building a TFN model with more than one input and how a modeller should cautiously use the identification tools of Section 17.3.1 for the multiple input case.

In Section 17.3.1, the following three identification procedures are explained in detail for the case of a TFN model with a single input:

(i) the empirical identification approach,

(ii) the Haugh and Box identification method, and

(iii) the Box and Jenkins identification procedure.

All three methods were specially developed under the assumption that only one input series is present in the model and the input series only affects the output. In fact, this assumption is theoretically embedded into the latter two procedures. When there is more than one input series, the obvious way to use each identification procedure is to investigate, pairwise, the relationship between each x_{ti} series and the y_t in order to design the form of the transfer function $v_i(B)$. However, in a TFN model with more than one covariate series, two or more covariates may not be independent of one another and may therefore affect each other in addition to driving the response variable. If there is not too much interaction among the x_{ti} series, fairly correct transfer functions may be identified using the pairwise identification procedure. Whatever the case, the assumption that the x_{ti} 's are independent of one another is not assumed for the TFN model itself in [17.5.3]. Therefore, if required, a number of tentative models can be fitted to the series. After also identifying the noise term, different discrimination techniques can be used to isolate the most appropriate model or set of models. For example, one can select the model having the lowest value of the AIC in [6.3.1] or the BIC in [6.3.5]. One can also remove any parameter from a model whose estimated value is not significantly different from zero. Finally, one should also insure that this model passes diagnostic checks, especially the tests for determining the whiteness of the estimated a_t series.

When designing the transfer function, the most suitable approach is probably to use the empirical approach in conjunction with the method of Haugh and Box. Although it may be complicated to use in practice, another procedure for identifying transfer functions is to employ the method of Liu and Hanssens (1982) which is specifically designed for identifying TFN models with multiple inputs. However, when deciding upon the parameters needed in the noise term, N_t, it is recommended for most applications that the empirical technique be used. Recall from Section 17.3.1 for the empirical approach, that after designing the transfer functions, the model in [17.5.3] is fitted to the set of series where it is initially assumed the noise term is white, even though it is probably not. Next, by following the model development phases of Chapters 5 to 7 for a single series, the best ARMA model is selected for modelling the estimated noise sequence form the previous step. Keep in mind, that because the noise term is assumed to be independent of the dynamic component, this is a theoretically valid procedure. Finally, the identified noise term can be used in [17.5.3] and then all of the model parameters can be simultaneously estimated for the completely identified TFN model.

Because the a_t's are assumed to be $NID(0,\sigma_a^2)$, MLE's can be efficiently calculated using the method of McLeod (1977) or another appropriate estimate such as one of those listed in Section 6.2.3. Not only are the MLE's obtained using McLeod's method almost exact MLE's, but the computation time required is much lower than that needed by other available exact MLE procedures. The estimation procedure described in Appendix A17.1 for a TFN with one input, can easily be expanded for use with a TFN model having multiple inputs.

At the model validation stage, the key assumption to check is that the residuals, $\hat{a}_t$, which are estimated along with the model parameters, are white. As explained in Section 17.3.3, this can be accomplished by investigating a plot of $r_{\hat{a}\hat{a}}(k)$ from [16.2.6] or [7.3.1] along with the 95% confidence limits. If there are problems, the form of the significantly large autocorrelations present in $r_{\hat{a}\hat{a}}(k)$ may indicate what type of model modifications should be made to either N_t, the dynamic component, or both. Investigating the form of the residual CCF between each prewhitened x_{ti} series and $\hat{a}_t$ may also assist in detecting where the sources of the problems are located and how they should be rectified. However, if the x_{ti} series were not previously prewhitened for use in a causality study (see Section 16.2.2) or some other purpose, obtaining the residual CCF pairwise for each prewhitened x_{ti} series and $\hat{a}_t$ may be quite time consuming. Furthermore, the alterations suggested by each individual residual CCF may not necessarily hold for the overall TFN model in [17.5.3] because of possible interactions among the x_{ti} series themselves. Fortunately, the authors have found in practice that when the empirical identification approach is utilized in conjunction with a sound understanding of the physical realities of the problem being studied, usually problems with the design of the TFN model can be circumvented.

17.5.4 Hydrometeorological Application

Introduction

The general procedure in many modelling problems is to start with a simple model and then increase the model complexity until an acceptable description of the phenomenon is achieved or until further improvements in the model cannot be obtained by increasing the model complexity. This is especially true in TFN modelling where there is a single response variable and multiple input series. However, the question arises as to how one can conveniently construct the most effective model for describing the dynamic relationships between the output and input series.

The purpose of the hydrometeorological application in this section is to demonstrate the *art and science of building a suitable TFN model*. As will be shown, a range of different TFN models are developed and the most appropriate model is systematically found. The output for each TFN model always represents the deseasonalized average monthly logarithmic flows of the Saugeen River at Walkerton, Ontario, Canada, while the covariate series consist of either transformed precipitation or temperature data sets, or both types of series. The types, lengths and locations of measurement for the data sets entertained are shown in Table 7.5.1 for the single riverflow sequence, the two precipitation and the two temperature series. The riverflow data are obtained from Environment Canada (1980a) and the precipitation and temperature data are provided by the Atmospheric Environment Service in Downsview Ontario (Environment Canada, 1980b).

Table 17.5.1. Available monthly data.

Type	Location	Period
Riverflows	Saugeen River at Walkerton, Ontario	1963-1979
Precipitation	Paisley, Ontario	1963-1979
Precipitation	Lucknow, Ontario	1950-1979
Temperature	Paisley, Ontario	1963-1979
Temperature	Lucknow, Ontario	1950-1979

When the data are used in the upcoming application as series in the CCF analyses or as input or output series in the TFN models, the time series are only employed for the time period during which all the series overlap. However, when estimating missing observations within a single time series, the entire time series is used (see Section 19.3). For a further discussion and the original presentation of this application, the reader may wish to refer to the paper of Hipel et al. (1982).

Missing Data

Prior to constructing a TFN model, any missing data in the covariate series must be estimated. The only missing data in this study are ten precipitation and corresponding temperature data points for the Lucknow station where the dates of these missing data are given in Table 16.5.2. As is explained in detail in Section 19.3 in the chapter on intervention analysis, a special type of intervention model can be designed for obtaining good estimates of the missing data points. An inherent advantage of the intervention analysis approach for estimating missing data points is that the correlation structure of the series is automatically taken into account when obtaining the estimates for the missing observations. When the intervention model in [19.3.7], developed in Section 19.3.6 for the entire deseasonalized Lucknow temperature data, is employed for data filling, the estimates and SE's given in the third column of Table 17.5.2 are obtained for the original series. The estimates for the missing observations can be compared to their respective monthly means in the second column of Table 17.5.2. For this particular application, the difference between each estimate and its monthly mean is always less than its SE.

Table 17.5.2. Estimates of missing temperature data at Lucknow.

Dates	Monthly Means (C°)	Estimates (C°) (SE's)
February 1953	-6.48	-6.57 (2.32)
May 1968	11.99	11.81 (1.78)
September 1968	15.17	15.34 (1.16)
October 1973	9.60	9.78 (1.67)
August 1975	18.89	18.59 (1.20)
September 1975	15.17	15.29 (1.16)
July 1976	19.68	19.47 (1.09)
September 1978	15.17	15.30 (1.16)
October 1978	9.60	8.30 (1.70)
August 1979	18.89	18.56 (1.18)

When the Lucknow precipitation data is deseasonalized using [13.2.3], the resulting nonseasonal sequence is white noise. Because there is no correlation structure in the series, the appropriate estimate of each missing data point is simply taken as its average monthly value.

The intervention analysis approach to data filling in Section 19.3 can be used when not more than about 10% of the data are missing. If there are many missing observations, where there may be rather long periods of time over which there are no data at all, the technique of Section 22.2 may be useful. Furthermore, when there are two series which are causally related but one is longer than the other, a TFN model relating the two series can be used to obtain estimates of the shorter series where it doesn't overlap with the longer one. This technique is called backforecasting and is explained in Section 18.5.2. Whatever the case, once the unknown observations have been estimated for each input series, a TFN model can be built for describing the relationships between the output and the inputs.

Identifying the Dynamic Component

Based upon a physical understanding of the problem and also using residual CCF analyses, the possible forms of the transfer functions can be identified for linking precipitation or temperature to the riverflow output. Firstly, the Saugeen River flows are transformed using a logarithmic transformation ($\lambda = 0$ in [3.4.30]) in order to avoid problems of non-normality and/or heteroscedasticity in the model residuals. It is found that the precipitation and temperature series do not require a power transformation. Next, an ARMA model is fitted to each deseasonalized series in Table 17.5.1 and the model residuals are estimated. Finally, the residual CCF between the residuals from the model fitted to the Saugeen River flows and each of the other four residual series are then calculated using [16.2.6].

The results of the cross-correlation analyses show a positive significant relationship at lag zero for each of the two precipitation series. For instance, the plot of the CCF for the Lucknow precipitation and Saugeen riverflows is displayed in Figure 17.5.1 along with the estimated 95% confidence interval. The value of the residual CCF at lag zero in Figure 17.5.1 is 0.448 whereas for the Paisley precipitation the estimated value of 0.365 is slightly smaller. Although the residual CCF plot for the Paisley precipitation is not shown, it is indeed similar in form to Figure

17.5.1. The characteristics of the residual CCF's for the two precipitation series makes intuitive sense from a physical point of view since for monthly data, most of the precipitation for a particular month will result in direct runoff in the same month.

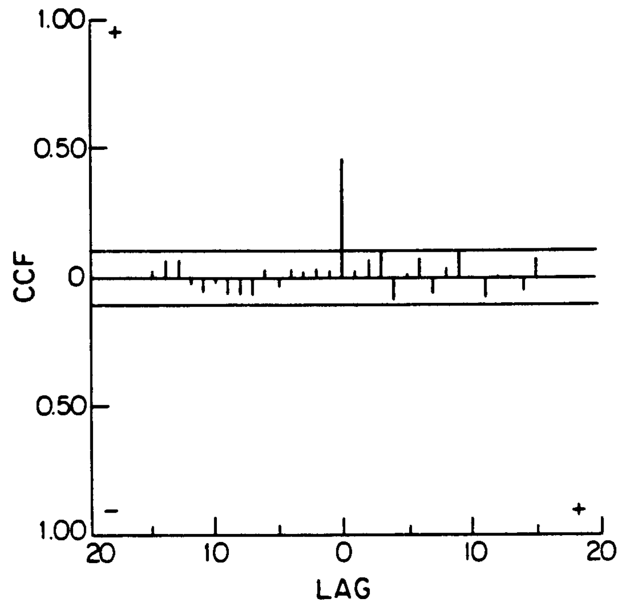

Figure 17.5.1. Residual CCF for the Lucknow precipitation and Saugeen riverflows.

The results of the residual CCF analyses for the two temperature series and the Saugeen riverflows are somewhat different. In these cases, there are no significant cross-correlations at any lag. However, from a physical viewpoint, one might expect that above average temperatures during the winter season would increase snowmelt and thus riverflow. For this reason, the temperature series are considered in the upcoming TFN model building. The temperature series are assumed to have a significant contribution at lag zero and as will be shown later, this assumption is found to be justifiable.

Combining Multiple Times Series

Often more than one covariate series of a particular kind is available to the analyst. In hydrological studies, data from several precipitation and temperature stations within or near the basin may be available. A common procedure employed by hydrologists to reduce model complexity is to combine similar types of series to form a single input covariate series. In the case of precipitation data, the records from the various stations are often combined to provide a single series of mean precipitation for a given region or basin. Two common methods of combining precipitation series are the Isohyetal and the Thiessen polygon techniques (Viessman et al., 1977). These procedures are essentially graphical methods and require a skilled analyst to obtain reasonable and consistent results. In an effort to automate procedures for combining similar types of series and provide more consistent results, a technique based on combining transfer function coefficients is presented.

Consider the case where two input covariate series, x_{t1} and x_{t2} , are to be combined to form a single input covariate series x_t . If x_{ti} causes y_t instantaneously, then the TFN models for the two series would be

$$y_t = \omega_{01}x_{t1} + N_{t1} \qquad [17.5.4]$$

and

$$y_t = \omega_{02}x_{t2} + N_{t2} \qquad [17.5.5]$$

where ω_{01} and ω_{02} are the transfer function parameters for the series x_{t1} and x_{t2} , respectively. For this example, the two series x_{t1} and x_{t2} would be combined using the relative ratio of the transfer function coefficients such that

$$\left[\frac{\omega_{01}}{\omega_{01}+\omega_{02}}\right]x_{t1} + \left[\frac{\omega_{02}}{\omega_{01}+\omega_{02}}\right]x_{t2} = x_t \qquad [17.5.6]$$

If more than two input series were available, this procedure could simply be extended to combine all of the available data into one input covariate series.

The Transfer Function-Noise Models

The various TFN models and one ARMA model that are examined, as well as their associated AIC values, are presented in Table 17.5.3. A decrease in the value of the AIC indicates that the additional model complexity is probably warranted since a better statistical fit is obtained. As expected, each increase in model complexity leads to a corresponding decrease in the value of the AIC. Thus a better description of the phenomena results with each addition of available information. For all of the TFN models in Table 17.5.3, the noise term is identified using the empirical approach of Section 17.3.1 to be an ARMA(1,0) model. Furthermore, all of the models satisfy the diagnostic checks presented in Section 17.3.3. Details of each of the models are now discussed.

The first model considers only the Saugeen riverflows. The time series is first transformed by taking natural logarithms of the data. This series is then deseasonalized by subtracting the estimated monthly mean and dividing by the estimated monthly standard deviation for each observation as in [13.2.3]. An ARMA(1,0) model is found to be the best model to fit to these deseasonalized flows. The value of the AIC is 963.121 and this value is used as a basis for comparing improvements in each of the subsequent models in Table 17.5.3.

Precipitation Series as Inputs: As suggested by the results of the residual CCF analyses, each of the precipitation series is used as an input covariate series. Prior to fitting the TFN model, each of the series is first deseasonalized. Each series is then used independently as an input covariate series in a TFN model. As shown in Table 17.5.3, the transfer function parameter, ω_0 , for Paisley and Lucknow are estimated as 0.310 and 0.350, respectively. Note that the AIC value for the Lucknow precipitation series is significantly less than the AIC value for Paisley. This may suggest that the pattern of the overall precipitation which falls on the Saugeen River basin upstream from Walkerton, is more similar to the precipitation at Lucknow than the precipitation at Paisley, even though Paisley is closer to Walkerton than Lucknow.

Table 17.5.3. Transfer function-noise models fitted to the data.

Output and Covariate Time Series	Parameter Estimates (SE's)	AIC
Saugeen Flows	$\hat{\phi}_1 = 0.407\ (0.064)$	963.121
Saugeen Flows Paisley Precipitation (ω_0)	$\hat{\phi}_1 = 0.405\ (0.064)$ $\hat{\omega}_0 = 0.310\ (0.055)$	936.129
Saugeen Flows Lucknow Precipitation (ω_0)	$\hat{\phi}_1 = 0.429\ (0.063)$ $\hat{\omega}_0 = 0.350\ (0.053)$	925.606
Saugeen Flows Combined Precipitation (ω_0)	$\hat{\phi}_1 = 0.418\ (0.064)$ $\hat{\omega}_0 = 0.350\ (0.054)$	926.340
Saugeen Flows Summer Precipitation (ω_{01}) Winter Precipitation (ω_{01})	$\hat{\phi}_1 = 0.407\ (0.064)$ $\hat{\omega}_{01} = 0.444\ (0.065)$ $\hat{\omega}_{02} = 0.183\ (0.092)$	922.270
Saugeen Flows Summer Precipitation (ω_{01}) Accumulated Snow (ω_{02})	$\hat{\phi}_1 = 0.408\ (0.064)$ $\hat{\omega}_{01} = 0.448\ (0.066)$ $\hat{\omega}_{02} = -0.162\ (0.109)$	924.037
Saugeen Flows Paisley Temperature (ω_0)	$\hat{\phi}_1 = 0.414\ (0.064)$ $\hat{\omega}_0 = 0.073\ (0.061)$	963.700
Saugeen Flows Lucknow Temperature (ω_0)	$\hat{\phi}_1 = 0.419\ (0.064)$ $\hat{\omega}_0 = 0.112\ (0.062)$	961.860
Saugeen Flows Summer Precipitation (ω_{01}) Winter Precipitation (ω_{02}) Lucknow Temperature (ω_{03})	$\hat{\phi}_1 = 0.422\ (0.064)$ $\hat{\omega}_{01} = 0.453\ (0.064)$ $\hat{\omega}_{02} = 0.194\ (0.090)$ $\hat{\omega}_{03} = 0.144\ (0.055)$	917.558
Saugeen Flows Summer Precipitation (ω_{01}) Winter Precipitation (ω_2) Combined Temperature (ω_{03})	$\hat{\phi}_1 = 0.420\ (0.064)$ $\hat{\omega}_{01} = 0.453\ (0.064)$ $\hat{\omega}_{02} = 0.194\ (0.090)$ $\hat{\omega}_{03} = 0.133\ (0.055)$	918.586

Using the procedure outlined in the previous section, the two precipitation series are combined to form a single input covariate series. In this study, the Lucknow and Paisley precipitation series are combined in the ratio 53:47, respectively. This combined precipitation series is then deseasonalized and used as an input series for the TFN model. The resulting AIC value is only slightly larger than the AIC value obtained when only the Lucknow precipitation series is employed. Since the difference is small, either model would be satisfactory and for the balance of this section the combined precipitation series is employed.

In the previous models, the precipitation series are entered as a single series having a single transfer function parameter. In these cases, it is therefore assumed that the contribution of precipitation is the same throughout the year. Physically, however, it makes sense that the contribution of precipitation during the winter months would be less than the contribution during the warmer periods of the year, since the precipitation accumulates on the ground in the form of snow during the cold season. In an effort to better reflect reality, the single precipitation series formed by combining the Lucknow and Paisley data, is divided into two separate seasons.

In separating the precipitation into two seasons, the winter season is taken as those months where the mean monthly temperature is below zero degrees Celcius. For both the Paisley and the Lucknow temperature series, December, January, February and March have mean monthly temperatures below freezing. Therefore, the winter precipitation series consists of the deseasonalized precipitations for these four months and zeros for the other eight months of the year. Conversely, the summer precipitation series has zeros for the four winter months and the deseasonalized precipitations for the remaining entries. These two series are input as separate covariate series with separate transfer function parameters. The resulting calibrated TFN model is

$$y_t = 0.444x_{t1} + 0.183x_{t2} + \frac{a_t}{(1-0.407B)} \qquad [17.5.7]$$

where y_t is the deseasonalized logarithmic riverflow at time t; x_{t1} is the combined deseasonalized summer precipitation series; and x_{t2} is the combined deseasonalized winter precipitation series. As expected, the transfer function coefficient for the summer precipitation is larger than the transfer function parameter for the winter precipitation. It is also reassuring to note that the better representation of the physics of the system also leads to an improved statistical fit as indicated by a lower AIC value.

A second type of dynamic model aimed at modelling the spring runoff resulting from snowmelt is also considered. In this model, the summer precipitation is the same as the model in [17.5.7]. However, the snow accumulated during the winter months from December to March is represented as a single pulse input in April where the temperature is above zero for the first time and hence spring runoff occurs. For the other eleven months of the year this series has zero entries. This type of dynamic model has been shown to work well for river systems located in areas that experience Arctic climate (Baracos et al., 1981) and rarely have any thaws during the winter months. However, the climatic conditions in the Saugeen River basin during the winter are not extremely cold and several midwinter melts result in a significant reduction in the accumulated snow cover on the ground. For this reason, the transfer function parameter for the accumulated winter precipitation is not significantly different from zero. Notice in Table 17.5.3 that that value of $\hat{\omega}_{02} = -0.162$ is much less than twice the SE of 0.109.

When dealing with quarter monthly data, another approach is presented in Section 18.3 and also by Thompstone (1983, Ch. 6) for incorporating snowmelt into a TFN model. The advantage of this approach is that it closely reflects the physical processes of snowmelt.

Temperature Series as Inputs: Although the residual CCF analyses indicate no significant relationships between temperature and riverflow, the two temperature series are used as input covariate series in TFN models. As before, both series are first deseasonalized by subtracting out the estimated monthly means and dividing by the estimated monthly standard deviations for each observation as in [13.2.3]. These series are then entered independently as covariate series in

TFN models. The resulting models and their associated AIC values are shown in Table 17.5.3. Because 1.96 times the SE for each parameter is larger than the parameter estimate, neither transfer function parameter is significant at the five percent significance level. Recall that the CCF for each temperature series and the Saugeen riverflows, also suggests that there may not be a marked relationship between the temperatures and riverflows. However, the Lucknow temperature parameter is significantly different from zero at the ten percent significance level. As a result, the Lucknow temperature series is included in the TFN models where both the temperature and precipitation series are included.

Precipitation and Temperature Series as Inputs: In an effort to combine all of the available information, both the temperature and the precipitation data are used as input covariate series in TFN models. In the first model of this type in Table 17.5.3, the combined precipitation is deseasonalized and split into two seasons as is done in [17.5.7]. The deseasonalized Lucknow temperature data is used as another input covariate series. The resulting model is given by

$$y_t = 0.453x_{t1} + 0.194x_{t2} + 0.144x_{t3} + \frac{a_t}{(1 - 0.422B)} \qquad [17.5.8]$$

where y_t is the deseasonalized logarithmic flow at time t; x_{t1} is the deseasonalized summer precipitation; x_{t2} is the deseasonalized winter precipitation; x_{t3} is the deseasonalized Lucknow temperature data; and a_t is the white noise term. The model and its associated AIC are also shown in Table 17.5.3. This model provides a significant improvement over any of the models previously employed with a decrease of almost five in the AIC when compared to the model in [17.5.7]. Also, the transfer function parameter for the temperature series is significantly different from zero in this case. Recall from before that the transfer function parameter for either temperature series is not significantly different from zero at the five percent significance level. However, when the precipitation series is included in the model, the temperature series provides a significant contribution. This point illustrates the need for more research in identifying the dynamic component of TFN models when more than one input covariate series is available.

The last model fitted to the data employs the combined precipitation and the combined temperature data. The temperature series are combined in the same fashion as the precipitation series but are not divided into two separate seasons. The resulting model and its associated AIC are shown in Table 17.5.3. Note that the AIC value is only marginally larger than that of the previous model where the Lucknow temperature data is employed instead of the combined temperature series. In this case, either of these last two TFN models could be employed as the most appropriate model for the available data.

17.6 ARMAX MODELS

From [17.2.7], the TFN model having a single covariate series is written as

$$y_t - \mu_y = \frac{\omega(B)}{\delta(B)} B^b (x_t - \mu_x) + \frac{\theta(B)}{\phi(B)} a_t \qquad [17.6.1]$$

The first and second terms on the right hand side of [17.6.1] are referred to as the dynamic and noise components in Sections 17.2.2 and 17.2.3, respectively. The operators contained in the transfer function $\frac{\omega(B)}{\delta(B)} B^b$ for transferring the influence of the covariate series x_t to the response series y_t are defined in Section 17.2.2. Finally, the operators in the transfer function $\frac{\theta(B)}{\phi(B)}$ for the white noise a_t are defined in [17.2.4] of Section 17.2.3 for the ARMA process describing the correlated or coloured noise as

$$N_t = \frac{\theta(B)}{\phi(B)} a_t \qquad [17.6.2]$$

Some authors prefer to write a dynamic model having a stochastic noise component in a different fashion than the one given in [17.6.1]. More specifically, their model is written as

$$\phi(B)(y_t - \mu_y) = \omega(B)B^b(x_t - \mu_x) + \theta(B)a_t \qquad [17.6.3]$$

In the literature, this model is most commonly called the *ARMAX (autoregressive-moving average-exogeneous variables) model* (Hannan, 1970) but is also referred to as the ARTF (autoregressive transfer function) and ARMAV models. The exogeneous variable in [17.6.3] refers to the input series, x_t. By dividing [17.6.3] by $\phi(B)$, the ARMAX model can be equivalently given as

$$(y_t - \mu_y) = \frac{\omega(B)}{\phi(B)} B^b(x_t - \mu_x) + \frac{\theta(B)}{\phi(B)} a_t \qquad [17.6.4]$$

A feature of the ARMAX model written in [17.6.3] is that the response variable is written directly as an autoregression. For example, if the order of $\phi(B)$ is 2, the model would be

$$y_t - \mu_y = \phi_1(y_{t-x} - \mu_h) + \phi_2(y_{t-2} - \mu_y) + \omega(B)B^b(x_t - \mu_x) + \theta(B)a_t$$

Nonetheless, the TFN model possesses distinct theoretical and practical advantages over the ARMAX model. To compare these two models consider the linear filter interpretations depicted in Figures 17.6.1 and 17.6.2 for the TFN model in [17.4.1] and ARMAX model in [17.6.4], respectively. In each figure, the inputs to the model are the a_t innovations plus the covariate x_t series. After passing through the indicated linear filters, the plus sign indicates that the dynamic and noise components are added together to create the y_t response. In both models, the input signal, x_t, and white noise, a_t, enter the linear filter system by different pathways. However, for the case of the ARMAX model in Figure 17.6.2, the input transfer function $\frac{\omega(B)}{\phi(B)}$ and the ARMA noise transfer function $\frac{\theta(B)}{\phi(B)}$ are interrelated, since they have the same common denominator $\phi(B)$. On the other hand, the transfer functions for the input and noise terms for the TFN model in Figure 17.6.1 have no common operators and are, therefore, independent of one another. Consequently, the TFN model is a more general representation of a dynamic-noise system than the ARMAX model (Young, 1984, pp. 113-116). The TFN model clearly separates out the deterministic or dynamic component and the stochastic noise effects.

As pointed out by Young (1984, pp. 116-117), one could also define other related versions of a dynamic-noise model outside of those given in [17.4.1] and [17.6.3]. For example, one could constrain the noise term in [17.6.1] or [17.6.4] to be purely AR or solely MA. Overall, the most general and flexible definition is the TFN model given in [17.6.1].

A great practical advantage of TFN modelling is that comprehensive model construction tools are available for conveniently applying TFN models to realworld problems. As explained in Section 17.3, flexible techniques are known and tested for use in identification, estimation and diagnostic checking of TFN models. However, this is not the situation for ARMAX models.

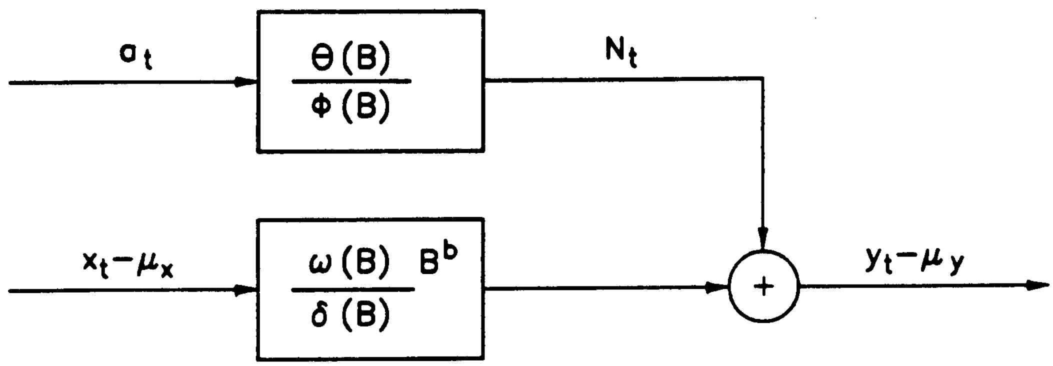

Figure 17.6.1. Linear filter depiction of the TFN model in [17.6.1].

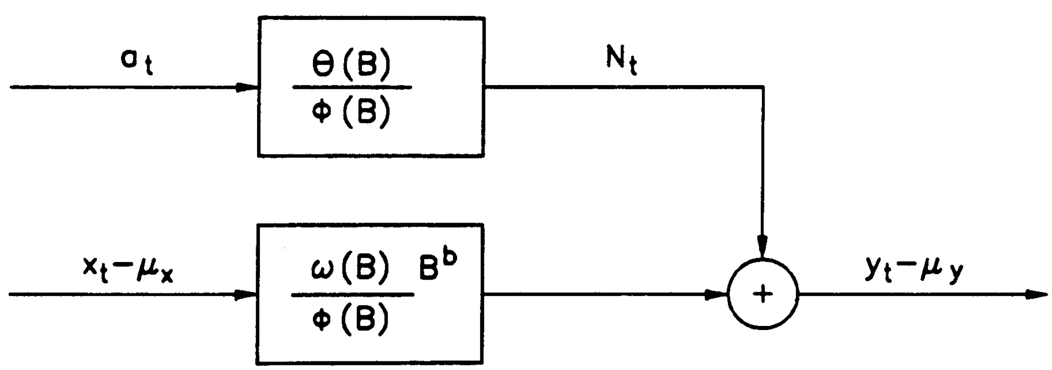

Figure 17.6.2. Linear filter interpretation of the ARMAX model in [17.6.4].

For example, one has to be careful when estimating the parameters of an ARMAX model. If the order of certain operators in the ARMAX model are not specified correctly, one may obtain poor parameter estimates (Young, 1984, p. 117). Because of the aforesaid and other reasons, it is recommended that the TFN model be selected over the ARMAX model for employment in practical applications.

The ARMAX model was introduced into system identification by Astrom and Bohlin (1965). In the past, ARMAX models have been successfully applied to hydrological and environmental engineering problems. For documented case studies of ARMAX modelling in hydrology, the reader can refer, for instance, to the research of Haltiner and Salas (1988), as well as references cited therein. In environmental engineering, ARMAX models have been employed for modelling dynamic systems problems arising in the conveyance (Capodoglio et al., 1990) and treatment (Capodoglio et al., 1991, 1992; Novotny et al., 1991) of wastewater. As a matter of fact, ARMAX and TFN models perform well when compared to more complex deterministic models written as differential equations, for capturing the key dynamic aspects of wastewater treatment plants (Capodoglio et al., 1992).

17.7 CONCLUSIONS

As exemplified by the applications in this chapter, a TFN model can be conveniently constructed for handling both single and multiple inputs. In addition to an understanding of the physical properties of a system being modelled, an array of well developed statistical tools are available for use in designing a suitable TFN model. For model identification, the empirical approach along with appropriate CCF analyses described in Section 17.3.1, can be used to identify the dynamic component of a TFN model. To identify the noise component of the TFN model, it is recommended that the empirical approach be used, especially when there are more than one covariate series. Subsequent to identifying one or more tentative TFN models, MLE's can be obtained for the model parameters (see Appendix A17.1) and a number of statistical tests can be employed for checking the validity of the model. Automatic selection criteria, such as the AIC and BIC, can be quite useful for model discrimination purposes.

If there are more than one precipitation or temperature series, a procedure is available for obtaining a single precipitation or temperature series. For the situation where snow accumulates during the winter time, the precipitation series can be incorporated into the dynamic model in specified manners so that the model makes sense from a physical point of view. For the case of the best Saugeen River dynamic model, the precipitation series was split into a winter and summer series, and a separate transfer function was designed for each of the series. An alternative approach for incorporating snowmelt into a TFN model is presented in Section 18.3.

When TFN models are fitted to other kinds of environmental series, the scientist can practice the *art and science* of his profession by designing physically based transfer functions and using flexible statistical tools to isolate the best design. As shown by the TFN applications in the next chapter, a properly designed TFN can produce accurate forecasts which in turn can be used in the control and operation of a system of reservoirs.

APPENDIX A17.1

ESTIMATOR FOR TFN MODELS

An estimator for obtaining MLE's for the parameters of a TFN model is outlined in this appendix. For convenience of explanation, the estimate is explained for the case of a TFN model having a single input series as in [17.2.5]. The expansion of this estimator for handling the situation where there are multiple input series as in [17.5.3], as well as interventions (see Chapter 19 and 22), is straightforward.

The estimator for a TFN model is, in fact, a direct extension of the estimator for an ARMA model. In general, the estimator works as follows. Recall from the start of Part VII that a TFN model can be qualitatively written as

single output = dynamic component + noise

where the dynamic component contains one or more input series and the correlated noise component is modelled as an an ARMA model process. As explained in Chapter 19, the dynamic component can also be designed for not only taking care of multiple input series but also the effects of external interventions upon the output series. Whatever the case, one can calculate the noise component as

noise = single output − dynamic component

Next, because the noise is assumed to be an ARMA process, one can calculate the white noise part for the correlated noise term by using an ARMA filter. Recall that these residuals are assumed to be NID($0,\sigma_a^2$). Finally, keeping in mind that one is simultaneously estimating the parameters for an overall TFN model, one can employ an ARMA estimator, such as the McLeod (1977) algorithm described in Appendix A6.1 or one of the other estimators listed in Section 6.2.3, to obtain MLE's for the model parameters. Possible optimization techniques for maximizing the likelihood function are also referred to in Section 6.2.3.

To be more specific, consider the TFN model in [17.2.5] having one input series, which is written as

$$
\begin{aligned}
y_t - \mu_y &= v(B)(x_t - \mu_x) + N_t \\
&= \frac{\omega(B)}{\delta(B)} B^b (x_t - \mu_x) + N_t \\
&= \frac{\omega(B)}{\delta(B)} (x_{t-b} - \mu_x) + N_t
\end{aligned}
\qquad [A17.1]
$$

where y_t is the output or response series having a theoretical mean of μ_x and x_t is the input or covariate series that has a theoretical mean of μ_x. The transfer function for the input series models the influence of x_t upon y_t and is given as

$$\frac{\omega(B)}{\delta(B)}B^b = \frac{(\omega_0 - \omega_1 B - \omega_2 B^2 - \cdots - \omega_m B^m)B^b}{(1 - \delta_1 B - \delta_2 B^2 - \cdots - \delta_r B^r)}$$

where b is a positive integer for modelling any delay in time for x_t to affect y_t. If there is no delay effect, then $b = 0$. Lastly, because N_t is assumed to follow an ARMA process as in [3.4.4],

$$\phi(B)N_t = \theta(B)a_t$$

$$\text{or } N_t = \frac{\theta(B)}{\phi(B)}a_t \qquad \text{[A17.2]}$$

where the ARMA filter is given as

$$\frac{\theta(B)}{\phi(B)} = \frac{(1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q)}{(1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p)}$$

and the a_t's are assumed to be NID$(0,\sigma_a^2)$.

Before fitting a TFN model to the data, the response and covariate series may undergo a transformation such as the Box-Cox transformation in [3.4.30]. Whatever the case, the theoretical means of the y_t and x_t series can be estimated by the sample means given as

$$\hat{\mu}_y = \bar{y}_t = \frac{1}{n}\sum_{t=1}^{n} y_t \qquad \text{[A17.3]}$$

and

$$\hat{\mu}_x = \bar{x}_t = \frac{1}{n}\sum_{t=1}^{n} x_t \qquad \text{[A17.4]}$$

respectively, where n is the number of observations in the x_t and y_t series. One could also simultaneously estimate μ_x and μ_y along with the other model parameters, but for the length of series that are usually analyzed, the sample means provide adequate estimates. The remaining parameters to be estimated in the transfer function in the dynamic component are

$$\delta = (\delta_1, \delta_2, \ldots, \delta_r)$$

$$\omega = (\omega_0, \omega_1, \omega_2, \ldots, \omega_m)$$

where ω_0 is always included as a parameter in any transfer function. The parameters which must be estimated in the noise term are

$$\phi = (\phi_1, \phi_2, \ldots, \phi_p)$$

$$\theta = (\theta_1, \theta_2, \ldots, \theta_q)$$

Because the a_t innovations are automatically estimated during the estimation procedure, in the final iteration the variance, σ_a^2, of the innovations can be calculated as

$$\hat{\sigma}_a^2 = \frac{1}{n}\sum_{t=1}^{n}\hat{a}_t^2 \qquad [A17.5]$$

where $\hat{a}_t$ is the estimate for a_t.

Based upon the normality assumption for the innovations, one can obtain the likelihood function for a model and employ an optimization procedure for iteratively finding the values of the parameter which converge to values that maximize the likelihood function. For a given maximum likelihood estimator, at each iteration one must be able to calculate the a_t's based upon the current values of the parameters as well as the structure of the model. As explained by Box and Jenkins (1976, p. 389), one can determine the current values of the a_t's for the TFN model in [A17.1], by following a three stage procedure. Firstly, the output, d_t, from the dynamic component can be computed from

$$d_t = \frac{\omega(B)}{\delta(B)}B^b(x_t - \bar{x}) = \frac{\omega(B)}{\delta(B)}(x_{t-b} - \bar{x})$$

or

$$\delta(B)d_t = \omega(B)(x_{t-b} - \bar{x})$$

or

$$d_t - \delta_1 d_{t-1} - \cdots - \delta_r d_{t-r} = \omega_0(x_{t-b} - \bar{x}) - \omega_1(x_{t-b-1} - \bar{x}) - \cdots - \omega_m x_{t-b-m} \qquad [A17.6]$$

Next the noise, N_t, can be determined using

$$N_t = (y_t - \bar{y}) - d_t \qquad [A17.7]$$

Thirdly, the a_t's can be obtained from [A17.2] as

$$a_t = \frac{\phi(B)}{\theta(B)}N_t$$

or

$$a_t = \theta_1 a_{t-1} + \theta_2 a_{t-2} + \cdots + \theta_q a_{t-q} + N_t - \phi_1 N_{t-1} - \phi_2 N_{t-2} - \cdots - \phi_p N_{t-p} \qquad [A17.8]$$

In order to calculate the innovations at each iteration in the overall optimization procedures, appropriate starting values are required for x_t's, y_t's and a_t's in [A17.6] to [A17.8], respectively. As noted by Box and Jenkins (1976, p. 389), the effects of transients can be minimized if the difference equations are initiated using a value of t for which all previous x_t's and y_t's are known. Consequently d_t in [A17.6] is computed from $t = u + 1$ onwards, where u is the larger of r and $m + b$ and d_t's occurring before $u + 1$ are set equal to zero. In turn, N_t in [A17.7] can be determined from N_{u+1} onwards. Finally, if the unknown a_t's are set equal to their unconditional expected values of zero, the a_t's can be calculated from a_{u+p+1} onwards.

Since one knows how to calculate the a_t's in the ARMA noise component, an estimator developed for an ARMA model can be employed within the TFN structure to obtain MLE's for the parameter of both the noise and dynamic components. For example, one could employ the estimator of McLeod described in Appendix A6.1. The standard errors of the estimated parameters δ, ω, ϕ and θ are obtained by inverting the observed information which is obtained by numerical diferentiation of the log likelihood function.

Brockwell and Davis (1987) give an alternative method for estimating TFN models using the Kalman filter. This technique yields exact maximum likelihood estimates as opposed to the method outlined above which is approximate. In practice, however, there is normally little difference between estimates produced by the two approaches, particularly when the lengths of the series exceed 50 data points.

PROBLEMS

17.1 Select two yearly series called x_t and y_t for which you think that x_t causes y_t and then carry out the following tasks:

(a) Following the procedure of Section 16.2.2, employ the residual CCF to ascertain if your suspected causality relationship between x_t and y_t is true.

(b) Employing the three identification approaches of Section 17.3.1, design a TFN model for formally connecting x_t and y_t. Which identification method provides the most clear results and is easiest to apply?

(c) After estimating the model parameters for the TFN identified in part (b), execute suitable diagnostic checks from Section 17.3.3 and make any necessary changes to the model. Be sure to explain all of your results and write down the difference equation for the final model.

17.2 By referring to the paper of Haugh and Box (1977), outline how these authors derive their identification procedure for a TFN model.

17.3 Explain the main steps that Box and Jenkins (1976) follow to derive their TFN identification method.

17.4 Based upon your experiences in fitting TFN models to data as well as theoretical attributes of the identification procedures, compare the three identification methods described in Section 17.3.1 according to advantages and limitations.

17.5 By referring to the literature in a field of interest to you, locate three articles describing applications of TFN models. Briefly outline the types of TFN applications carried out in the papers and chapters and how the TFN modelling was of assistance to the authors.

17.6 Follow the instructions of problem 17.1 for the situation where you employ average monthly or other types of seasonal data.

17.7 Construct a TFN model for formally modelling a yearly output series for which you have two meaningful annual covariate series. For instance, you may have an average annual riverflow series as well as yearly precipitation and temperature records.

17.8 Carry out the instructions of problem 17.7 for the case of monthly or quarter monthly time series.

17.9 Obtain an average monthly riverflow data set for which you have at least two precipitation and two temperature series. Follow the approach of Section 17.5.4, to systematically select the most appropriate TFN model to link your data sets.

17.10 After reading the paper of Haltiner and Salas in which they employ the ARMAX model of Section 17.6 for short-term forecasting of snowmelt runoff, do the following:

(a) Outline the approach that they employ for modelling how streamflow is affected by other hydrological variables.

(b) Compare the procedure of Haltiner and Salas for modelling runoff to that given in Section 17.5.4.

(c) Explain how Haltiner and Salas could employ a TFN model instead of an ARMAX model to formally model their hydrological data sets.

17.11 Capodaglio et al. (1992) demonstrate that ARMAX or TFN models perform as well as deterministic differential equations for describing certain wastewater treatment processes. For their applications, explain how they accomplished this. Find and explain another physical systems problem where TFN models fare as well or better than their deterministic counterparts.

17.12 The TFN model is an example of a finite difference equation that mathematically models the relationships among data sets available at discrete time points. In continuous time, one employs stochastic differential equations. Explain the continuous-time versions of the TFN models in [17.2.5] and [17.5.3].

17.13 As pointed out in Section 17.1, Delleur (1986) demonstrates that a TFN is physically justified for modelling flows in a watershed. By referring to Delleur's paper and using both differential and difference equations, explain how he does this.

17.14 Find a paper in a field which is of interest to you that clearly explains the relevance of TFN modelling for describing physical and/or socio-economic systems. Summarize the main findings of the paper and be sure to emphasize which results you think are most interesting.

17.15 By referring to papers such as those by Novotny and Zheng (1989) and Capodaglio et al. (1990), explain how TFN models can be employed for approximately modelling nonlinear relationships between variables.

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For other references related to TFN modelling, the reader may wish to refer to Chapters 16, 18, 19, 20 and 22.

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