Journal of Water Resources Planning and Management

Volume 113 Number 1 January 1987

TECHNICAL PAPERS

TECHNICAL NOTE

COMBINING HYDROLOGIC FORECASTS

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ABSTRACT: Forecasts of river flows are useful in optimizing the operation of multipurpose reservoir systems. Using two case studies, the usefulness of combination techniques for improving forecasts is examined. In the first study, a transfer function-noise model, a periodic autoregressive model, and a conceptual model are employed to forecast quarter-monthly river flows. These models all approach the modeling and forecasting problem from three different perspectives, and each has its own particular strengths and weaknesses. The forecasts generated by the individual models are combined in an effort to exploit the strengths of each model. The results of this case study indicate that significantly better forecasts can be obtained when forecasts from different types of models are combined. In the second study, periodic autoregressive models and seasonal autoregressive integrated moving average models are used to forecast monthly river flows. Combining the individual forecasts from these two statistical time series models does not result in significantly better forecasts.

INTRODUCTION

In the design and operation of engineering projects, such as a system of multipurpose reservoirs, it is both desirable and necessary to ensure optimum performance of the system. Prior to actual construction, simulated inputs can be employed to estimate the expected benefits and costs associated with the project. During operation, accurate forecasts of the system's inputs are necessary to ensure that the system is operated in the most efficient manner. Choosing a model which produces reliable and accurate forecasts is therefore essential to the successful operation of the system.

The selection of the "best" forecasting procedure is certainly a goal of any forecasting study. Invariably, however, no one method will produce optimum forecasts in all cases. The task then becomes one of selecting the most appropriate forecasting procedure based upon the available information.

An alternative approach is to combine the forecasts from two or more procedures in accordance with their relative performance. In this way, it is hoped that the strengths of each method might be exploited. The successes achieved by combining economic forecasts have been documented in several studies (Armstrong and Lusk 1983; Bates and Granger

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Note.—Discussion open until June 1, 1987. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on January 2, 1986. This paper is part of the *Journal of Water Resources Planning and Management*, Vol. 113, No. 1, January, 1987. ©ASCE, ISSN 0733-9496/87/0001-0029/$01.00. Paper No. 21147.

1969; Bordley 1982; Granger and Ramanathan 1984; Makridakis, et al. 1982; Newbold and Granger 1974; Winkler and Makridakis 1983). However, equivalent studies employing hydrologic time series are not available. Accordingly, the question of combining hydrologic forecasts is addressed in this paper.

Two case studies are presented. In the first experiment, a transfer function-noise (TFN) model, a periodic autoregressive (PAR) model and a conceptual hydrologic model are employed to produce one-step-ahead quarter-monthly river flow forecasts. These models all approach the modeling and forecasting problem from different perspectives, and each has its own particular strengths and weaknesses. The forecasts generated by the individual models are combined in an effort to exploit the strengths of each model.

In the second forecasting experiment, two specific statistical time series models are employed to forecast monthly river flows. The forecasts from these models are then combined using various procedures.

The specific types of models employed in the two seasonal forecasting case studies are outlined in the next section. A number of procedures for combining the individual forecasts are then presented. Finally, the results of a case study involving quarter-monthly river flows and a case study employing monthly river flows are discussed.

Forecasting Models

Periodic Autoregressive (PAR) Models.—Seasonality of geophysical data adds a degree of complexity to the selection and development of an appropriate model to fit to a given time series. Natural geophysical data are seasonally stationary. That is, river flows or temperatures for a particular season of the year are statistically similar from year to year, but may vary drastically across seasons. Hydrologists have also found that geophysical time series exhibit an autocorrelation structure which depends not only on the time lag between observations but also the season of the year (Moss and Bryson 1974). For example, if snowmelt is an important factor in runoff which might occur in either March or April, the correlation between observed river flows for these months may be negative, whereas it is usually positive at other times of the year. These characteristics offer a challenge to the practitioner while at the same time providing a convenient structure for the modeler to exploit. A family of PAR models specifically designed to account for these characteristics is briefly described in this section. A more detailed explanation of PAR models is given by Noakes (1984) and Noakes, et al. (1985).

Let Z_t, $t = 1, 2, \ldots,$ be a seasonal time series with period s. The time index t may be regarded as a function of the year T ($T = 1, 2, \ldots, N$), and the season m ($m = 1, 2, \ldots, s$). Thus, the time index may be written as $t = (T - 1)s + m$. The PAR ($p_1, p_2, \ldots, p_s$) model is conveniently described by

$$\phi^{(m)}(B)(Z_t^{(\lambda)} - \mu_t) = a_t \qquad (1)$$

where $\phi^{(m)}(B) = 1 - \phi_1^{(m)}B - \ldots - \phi_{p_m}^{(m)}B^{p_m}$ is the autoregressive (AR) operator for season m; B is the backward shift operator such that $BZ_t = Z_{t-1}$ and $B^m Z_t = Z_{t-m}$; $\mu_t = \mu^{(m)}$ is the mean for period m; and a_t is approximately NID$[0, \sigma^{2(m)}]$. The superscript m obeys modulo arithmetic (i.e., $\mu^{(1)} \equiv \mu^{(s+1)} \equiv \mu^{(-s+1)}$). The superscript λ is the exponent of an appropriate Box-Cox transformation. The Box-Cox transformation (Box and Cox 1964) is given by

$$Z^{(\lambda)} = \begin{cases} \lambda^{-1}[Z_t + \text{constant} - 1]; & \lambda \neq 0 \\ \ln(Z_t + \text{constant}); & \lambda = 0 \end{cases} \quad \ldots\ldots (2)$$

The reasons for transforming the data include stabilizing the variance and improving the normality assumption of the white noise series a_t. In hydrologic application, a Box-Cox transformation will often rectify problems associated with the untransformed series. In particular, a logarithmic transformation of the data is common in the analysis of hydrologic time series. For the river flow data employed in this study, the log transformation was found to be best.

Several procedures are available for determining the orders of the AR models to be fitted to each season (Noakes, et al. 1985). In this paper, plots of the seasonal partial autocorrelation functions (PACF) were examined to determine the orders of each seasonal AR model. Estimates of the model parameters were then obtained by solving the seasonal Yule-Walker equations. This model will be referred to as the PAR/PACF model (Noakes, et al. 1985).

Transfer Function-Noise (TFN) Models.—Detailed developments of the general TFN model are given in a number of publications (Box and Jenkins 1970; Box and Tiao 1975; Hipel, et al. 1975; Hipel and McLeod 1987; Hipel, et al. 1977). To be brief, only the general form of the model and the actual TFN model employed in the study are presented in this paper.

The general TFN model may be written in the form (Box and Jenkins 1970; Box and Tiao 1975; Hipel, et al. 1975; Hipel and McLeod 1987; Hipel, et al. 1977)

$$y_t = f(\underline{k}, \underline{x}, t) + N_t \quad \ldots\ldots (3)$$

where t is discrete time; y_t is the response variable; N_t is the stochastic noise component, which may be autocorrelated; and $f(\underline{k}, \underline{x}, t)$ is the dynamic component of y_t. The dynamic term includes a set of parameters $\underline{k}$ and a group of covariate series $\underline{x}$. When required, both y_t and x_t may be transformed using a suitable Box-Cox transformation. The same Box-Cox transformation need not be applied to all of the series.

Once a given series has been transformed, it is necessary to remove any trends or seasonality in the data. A common procedure employed in hydrology for monthly sequences is to deseasonalize the series by subtracting the estimated monthly mean and dividing by the estimated monthly standard deviation for each data point (Yevjevich 1972).

Two input covariate series were employed in the TFN model developed in this paper. These series consisted of deseasonalized rainfall and snowmelt data. The output from the model was deseasonalized logarithmic river flows. The TFN model had the form (Thompstone 1983)

$$y_t = \nu_1(B)x_{t,1} + \nu_2(B)x_{t,2} + N_t \quad \ldots\ldots (4)$$

where y_t is the transformed deseasonalized flow at time t; $x_{t,1}$ is the de-

TABLE 1.—Parameter Estimates for TFN Model

	PARAMETERS								
	$\delta_{1,j}$		$\omega_{0,j}$		$\omega_{1,j}$				
Calculations (1)	Rainfall series, $j = 1$ (2)	Snowmelt series, $j = 2$ (3)	Rainfall series, $j = 1$ (4)	Snowmelt series, $j = 2$ (5)	Rainfall series, $j = 1$ (6)	Snowmelt series, $j = 2$ (7)	ϕ_1 (8)	ϕ_2 (9)	θ_1 (10)
Estimated value	0.6248	0.5793	0.2326	0.1023	−0.2690	−0.0460	1.3112	−0.3817	0.7124
Associated standard error	0.0328	0.0895	0.0159	0.0131	0.0178	0.0168	0.1231	0.0916	0.1129

seasonalized rainfall at time t; and $x_{t,2}$ is the deseasonalized snowmelt at time t. The transfer functions $\nu_j(B)$ were identified as (Thompstone 1983)

$$\nu_j(B) = \frac{(\omega_{0,j} - \omega_{1,j}B)}{(1 - \delta_{1,j}B)}; \quad j = 1, 2 \quad \text{(5)}$$

where $\omega_{0,j}$, $\omega_{1,j}$ and $\delta_{1,j}$ are the model parameters which must be estimated from the data. The stochastic noise component was identified as an AR moving average (ARMA) process of the form (Thompstone 1983)

$$N_t = \frac{(1 - \theta_1 B)}{(1 - \phi_1 B - \phi_2 B_2)} a_t \quad \text{(6)}$$

where θ_1 is the first order moving average (MA) parameter; ϕ_i is the ith AR parameter; and a_t is white noise assumed to be NID$(0, \sigma_a^2)$. Estimates of the model parameters and their associated standard errors are given in Table 1.

Conceptual Hydrologic Model.—The conceptual model developed in hydrology attempts mathematically to model the complex physical processes involved in the hydrological cycle. The conceptual model employed in this paper was originally developed by S. I. Solomon and Associates (1974), and subsequently modified by Kite (1978). The model is a lumped parametric conceptual model and simulates the relationship between river flow and daily meteorological data. Inputs consist of total daily precipitation, minimum and maximum daily temperatures, and the previous day's flow.

Kite (1978) has previously compared this model with several larger well-known hydrologic models and found its performance comparable with the more complex models. With minor modifications, this model has been employed by Alcan Limited in Quebec to produce forecasts of daily river flows into their reservoir system (Thompstone 1983). Alcan's model is referred to as the PREVIS model in this paper.

Thompstone (1983) has noticed that the daily forecasts from the PREVIS model tend to be consistently higher or lower than the observed flows during certain periods. In an effort to compensate for this behavior, a crude smoothing has been applied to the PREVIS forecasts. A correction factor based on the average forecast error for the past few days is applied to the PREVIS forecasts. These smoothed forecasts are referred to as PREVIS/S forecasts.

Seasonal Autoregressive Integrated Moving Average (SARIMA Models).—The linear stochastic models described in the book by Box and Jenkins (1970) constitute a class of processes which is flexible and comprehensive enough to be useful in a wide range of applications. The success of these models has led to a corresponding increase in their popularity and subsequent use for modeling hydrologic time series (Hipel and McLeod 1987).

The SARIMA model for a discrete equispaced set of measurements Z_1, Z_2, ..., Z_n can be written as

$$\phi(B)\Phi(B^s)(1 - B)^d(1 - B^s)^D(Z_t - \mu) = \theta(B)\Theta(B^s)a_t \qquad (7)$$

where t is discrete time; s is the number of seasons per year; μ is the mean level of the process Z_t; and a_t is the noise component of the stochastic model assumed to be NID$(0, \sigma_a^2)$. The terms $\phi(B)$ and $\Phi(B^s)$ are the nonseasonal AR operator of order p and the seasonal AR operator of order P, respectively. Similarly, $\theta(B)$ and $\Theta(B^s)$ are the nonseasonal and seasonal MA operators of order q and Q, respectively. The nonseasonal differencing operator $(1 - B)^d$ is of order d, and the series is seasonally differenced D times using $(1 - B^s)^D$. The SARIMA models are usually represented by the notation $(p,d,q)(P,D,Q)$, where the first set of parentheses contains the orders of the nonseasonal operators, and the second set the orders of the seasonal operators.

Combining Forecasts

There are certainly countless ways of combining forecasts from different forecasting procedures to arrive at a combined forecast. The simplest is probably to weigh each forecast equally. If there are k forecasts available, the combined forecast f_c would be

$$f_c = \sum_{i=1}^{k} w_i f_i \qquad (8)$$

where f_i is the forecast produced by the ith model; w_i is the weighting factor for the ith forecast; and $w_i = w_j = 1/k$ for all i and j.

It would be expected that a "better" combination of forecasts could be obtained if the statistical properties of the forecast errors were considered. Winkler and Makridakis (1983) point out that if the covariance matrix of the forecast errors from k methods ($\boldsymbol{\Sigma}$) is known, then the optimal weights are given by

$$w_i = \frac{\sum_{j=1}^{k} \alpha_{ij}}{\sum_{h=1}^{k}\sum_{j=1}^{k} \alpha_{hj}} \qquad (9)$$

where the α_{ij} terms are the elements of $\boldsymbol{\Sigma}^{-1}$. In practice, $\boldsymbol{\Sigma}$ is not known and must be estimated. Estimates of the weights in Eq. 9 can be calculated from the inverse of $\hat{\Sigma}$, where

$$\hat{\Sigma}_{ij} = \nu^{-1} \sum_{h=t-\nu}^{t-1} e_h^{(i)} \cdot e_h^{(j)} \qquad (10)$$

$e_t^{(i)}$ is the percentage error for method i at time t; and ν is the number of previous forecast errors employed to calculate w_i.

In the study concerning the combination of economic forecasts by Winkler and Makridakis (1983), these authors found that estimating Σ^{-1} and calculating the weights using Eq. 9 gave the poorest results. One of the preferred procedures in their study was to ignore the correlation between the forecast errors. In this case the forecast weights were calculated as

$$\hat{w}_i = \frac{\left[\sum_{h=t-\nu}^{t-1} e_h^{(i)2} \right]^{-1}}{\sum_{j=1}^{p} \left[\sum_{h=t-\nu}^{t-1} e_h^{(j)2} \right]^{-1}} \qquad (11)$$

where $e_t^{(i)}$ and ν are as defined previously. This approach ensures that all of the estimated weights are greater than or equal to zero.

An alternative approach to calculating the combining weights when seasonal data are considered is presented in this paper. In this procedure, the model residuals are employed to calculate the residual variance for each season. If two forecasts are to be combined, then the weights are calculated for each season such that

$$w_{1,j} = \frac{\sum_{k=1}^{N} [a_{j+(k-1)s}^{(2)}]^2}{\sum_{k=1}^{N} [a_{j+(k-1)s}^{(1)}]^2 + \sum_{k=1}^{N} [a_{j+(k-1)s}^{(2)}]^2} \qquad (12)$$

and

$$w_{2,j} = \frac{\sum_{k=1}^{N} [a_{j+(k-1)s}^{(2)}]^2}{\sum_{k=1}^{N} [a_{j+(k-1)s}^{(1)}]^2 + \sum_{k=1}^{N} [a_{j+(k-1)s}^{(2)}]^2} \qquad (13)$$

where $w_{1,j}$ is the weight assigned to forecasting procedure 1 for the jth season; $w_{2,j}$ is the weight assigned to forecasting procedure 2 for the jth season; $a_t^{(i)}$ is the residual at time t for the ith model; N is the number of years of data; and s is the number of seasons per year. Since the data are seasonal, the forecast error variance might be expected to be seasonal, and it is hoped that this procedure will account for this seasonality.

Combining Quarter-Monthly River Flow Forecasts

The Lac St. Jean reservoir is part of the hydroelectric system operated by Alcan Limited in Québec. The electricity generated by this system is

TABLE 2.—RMSEs of Quarter-Monthly Forecasts Produced by Individual Models

Model (1)	RMSE (2)
TFN	0.2790
PAR/PACF	0.3009
PREVIS	0.3894
PREVIS/S	0.3537

used at Alcan's aluminum smelter in Arvida, Québec. In order to ensure a constant and adequate supply of power, it is necessary to schedule releases from the reservoir in an optimum fashion. Thus, forecasts of the quarter-monthly inflows into the reservoir are required so that the desired outflow and hydraulic head are available for power generation.

The available data in this study were 30 yrs of quarter-monthly river flows from 1953 to 1982 inclusive. A split sample experiment was conducted with models being calibrated using the first 27 yrs of data (Thompstone 1983). These models were then employed to generate forecasts for the last three years of data. The river flow data employed in this study were transformed using a logarithmic transformation (Thompstone 1983).

The RMSE's (rms error) of the logarithmic forecast errors for the individual models are presented in Table 2. The TFN model had the smallest RMSE of all the models considered. As such, this value will be used as a basis of comparison of the various techniques employed to combine the individual forecasts.

In the study, the weights for combining the individual forecasts were calculated using both Eq. 9 and Eq. 11 with $\nu = 4$, 8, and 12. Since the model residuals were not employed, the first ν forecasts were combined using equal weights. The weights were then recalculated for each subsequent forecast using the previous ν forecast errors.

The forecasts were combined in a pairwise fashion, with the exception of the two conceptual models (PREVIS and PREVIS/S). The resulting RMSE's of the combined forecasts, using Eq. 9 to calculate the combining weights, are given in Table 3. The subscripts associated with the RMSE indicate the number of previous forecast errors that were employed to calculate the weights. For example, when the previous four forecast errors were used to combine the TFN and PAR/PACF forecasts, the resulting RMSE was 0.1418. In most cases, the greater the number

TABLE 3.—RMSEs of Combined Quarter-Monthly Forecast with Combining Weights Calculated Using Eq. 9

Combination (1)	$RMSE_4$ (2)	$RMSE_8$ (3)	$RMSE_{12}$ (4)
TFN + PAR/PACF	0.1418	0.1197	0.1195
TFN + PREVIS	0.7868	0.5243	0.4183
TFN + PREVIS/S	0.9942	0.3183	0.2708
PAR/PACF + PREVIS	0.2432	0.2288	0.2224
PAR/PACF + PREVIS/S	0.2171	0.1858	0.1865

TABLE 4.—RMSEs of Combined Quarter-Monthly Forecasts with Combining Weights Calculated Using Eq. 11

Combination (1)	$RMSE_4$ (2)	$RMSE_8$ (3)	$RMSE_{12}$ (4)
TFN + PAR/PACF	0.1456	0.1236	0.1216
TFN + PREVIS	0.2751	0.2508	0.2474
TFN + PREVIS/S	0.2825[a]	0.2523	0.2498
PAR/PACF + PREVIS	0.2442	0.2298	0.2222
PAR/PACF + PREVIS/S	0.2144	0.1872	0.1884

[a]Larger RMSE than TFN forecast errors.

of previous forecast errors employed to calculate the weights, the smaller the resulting combined RMSE.

The smallest RMSE was obtained when the TFN and PAR/PACF forecasts were combined using the previous 12 forecast errors to calculate the weights. The resulting RMSE was less than half the value for the smallest RMSE for the individual models, suggesting that significant benefits could be obtained by combining the forecasts from these two models. Conversely, the largest RMSEs were obtained when the TFN forecasts were combined with the PREVIS or PREVIS/S forecasts. Only when the previous 12 forecast errors were employed to calculate the weights did the combined TFN and PREVIS/S forecasts yield a smaller RMSE than the best individual model. Even then, the difference was only in the third decimal place.

The resulting RMSE's of the combined forecasts when Eq. 11 was employed to calculate the combining weights are given in Table 4. In this case, only one combination had a larger RMSE than the best individual model. Once again, the smallest RMSE was obtained when the TFN and PAR/PACF forecasts were combined using the previous 12 forecast errors to calculate the combining weights. The largest RMSE's were obtained when the TFN forecasts were combined with the forecasts from the two conceptual models. These RMSE's did, however, represent a significant improvement when compared to the RMSE's obtained when Eq. 9 was used to calculate the combining weights.

As a test of combining forecasts from more than two models, the forecasts produced by the TFN, PAR/PACF, and PREVIS/S models were combined using equal weights. The resulting RMSE was 0.1355. Although this does not represent the lowest RMSE, even this naive combination of forecasts produced a RMSE which was less than half the RMSE of the best individual model.

Combining Monthly River Flow Forecasts

The data used in this study comprised 30 monthly unregulated river flow time series ranging in length from 37 to 68 yrs. The rivers are from a number of different physiographic regions, and vary in size from rivers with mean annual flows of less than 2 m^3/s to rivers with mean annual flows exceeding 100 m^3/s. The data for the Canadian rivers were obtained from Water Survey of Canada records, and the American river

TABLE 5.—Monthly River Flow Time Series

River (1)	Location (2)	Period (3)	$12N$ (4)	Mean annual flow (m^2/s) (5)
American	Fair Oaks, California	1906–60	660	106.18
Boise	Twin Springs, Idaho	1912–60	588	33.40
Clearwater	Kamish, Idaho	1911–60	600	231.31
Columbia	Nicholson, British Columbia	1933–69	444	108.61
Current	Van Buren, Missouri	1922–60	468	53.86
West Branch Delaware	Hale Eddy, New York	1916–60	540	30.32
English	Sioux Lookout, Ontario	1922–77	660	123.31
Feather	Oroville, California	1902–77	708	166.51
James	Buchanan, Virginia	1911–60	600	68.83
Judith	Utica, Montana	1920–60	492	1.44
Mad	Springfield, Ohio	1915–60	552	13.92
Madison	West Yellowstone, Montana	1923–60	456	12.88
McKenzie	McKenzie Bridge, Oregon	1911–60	600	46.94
Middle Boulder	Nederland, Colorado	1912–60	588	1.53
Missinaibi	Mattice, Ontario	1921–76	672	103.45
Namakan	Lac La Croix, Quebec	1923–77	648	107.46
Neches	Rockland, Texas	1914–60	564	68.53
N. Magnetawan	Burke Falls, Ontario	1916–77	732	5.53
Oostanaula	Resca, Georgia	1893–1960	816	78.10
Pigeon	Middle Falls, Ontario	1924–77	636	14.26
Rappahannock	Fredericksburg, Virginia	1908–71	768	45.31
Richelieu	Fryers Rapids, Quebec	1932–77	468	331.46
Rio Grande	Furnas, Minas Gerais, Brazil	1931–78	576	896.47
Saint Johns	Fort Kent, New Brunswick	1927–77	600	29.92
Saugeen	Walkerton, Ontario	1915–76	744	68.10
S.F. Skykomish	Index, Washington	1923–60	456	278.10
S. Saskatchewan	Saskatoon, Saskatchewan	1911–63	624	272.10
Trinity	Lewiston, California	1912–60	588	46.67
Turtle	Mine Center, Ontario	1921–77	672	37.08
Wolf	New London, Wisconsin	1914–60	564	49.34

flow series are from the United States Geological Survey. The Brazilian data were obtained from Eletrobras (the national electric company of Brazil). The rivers and their mean annual flows for the water year (October–September) are displayed in Table 5.

The general procedure was first to truncate the transformed data sets by omitting the last 36 observations. All the data in this study were transformed by taking natural logarithms, which was found to be the most appropriate Box-Cox transformation (Box and Cox 1964). The log transformation was needed to ensure that the model residuals were approximately normally distributed and homoscedastic.

Using the appropriate identification, estimation, and diagnostic check stages of model development, SARIMA and PAR models were fitted to the truncated series. The orders of the autoregressive models fitted to each season for the PAR models were determined by examining plots of the seasonal partial autocorrelation functions (PACF) (Noakes, et al.

TABLE 6.—Percentage of Times Model A Gave Values Better than Model B

	Model B						
Model A (1)	PAR/PACF (%) (2)	SARIMA (%) (3)	CMB-SEAS (%) (4)	CMB-3 (%) (5)	CMB-6 (%) (6)	CMB-9 (%) (7)	CMB-12 (%) (8)
PAR/PACF	0	70	56.7	60	60	56.7	56.7
SARIMA	30	0	20	16.7	20	20	20
CMB-SEAS	43.4	80	0	46.7	53.3	56.7	56.7
CMB-3	40	83.3	53.3	0	50	46.7	33.3
CMB-6	40	80	46.7	50	0	43.3	26.7
CMB-9	43.3	80	43.3	53.3	56.7	0	30
CMB-12	43.3	80	43.3	66.7	73.3	70	0

1985). These models, and the forecasts produced by them, are subsequently referred to as PAR/PACF models and forecasts in this paper. The SARIMA and PAR/PACF models were then used to generate 36 one-step-ahead forecasts for the 30 series.

The monthly forecasts produced by the PAR/PACF and SARIMA models were combined using some of the procedures outlined previously in this paper. The combining weights were calculated using Eq. 11 with $\nu = 3, 6, 9$, and 12. In addition, seasonal combining weights were calculated using Eqs. 12 and 13. The combined forecasts were compared on the basis of MSE's (mean squared errors).

A summary of the results is presented in Table 6. The CMB-SEAS entries refer to the combined forecasts produced when separate weights were calculated for each season. The CMB-ν entries refer to the combined forecasts when the previous ν forecast errors were employed to calculate the combining weights. The results show that, in general, the combined forecasts were not an improvement over the PAR/PACF forecasts regardless of the procedure employed to calculate the combining weights. Conversely, the SARIMA forecasts were almost always improved by combining them with PAR/PACF forecasts. A comparison of the various procedures for combining the forecasts seems to indicate that the more information employed to estimate the combining weights, the better the forecasts.

Conclusions

Combining economic forecasts from various models has become common practice. However, the case studies presented in this paper represent the first reported experiments dealing with the combination of river flow forecasts. Since even modest improvements in forecast performance provide the potential for significant increases in social and economic benefits, the results presented in this paper should be of particular interest to individuals operating and managing water resources systems.

TFN, PAR, and conceptual hydrologic models were employed to forecast quarter-monthly river flows in the first study. The results are far from conclusive since only one river was considered in the experiment.

The results are, however, encouraging. It appears that significant improvements in forecast performance can be achieved by combining forecasts produced by different types of models. The potential gains associated with improved forecasts should certainly be incentive enough to encourage further research in this area.

SARIMA and PAR/PACF models were employed to forecast monthly river flows in the second experiment. Individually, the PAR/PACF forecasts were, in general, significntly better than the SARIMA forecasts (Noakes, et al. 1985). The combined forecasts did not represent a significant improvement over the forecasts produced by the PAR/PACF models alone. Thus, although the SARIMA model forecasts seasonal economic data well, this model may not be appropriate for modeling and forecasting monthly river flows. The PAR model appears to be a more suitable model for monthly river flows (Noakes, et al. 1985).

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Appendix II.—Notation

The following symbols are used in this paper:

a_t = white noise component at time t assumed to be normally independently distributed with mean zero and constant variance;
$a_t^{(i)}$ = residual at time t for model i;
B = backward shift operator;
D = order of the seasonal differencing operator;
d = order of the nonseasonal differencing operator;
$e_t^{(i)}$ = percentage error for forecasting method i at time t;
$f(\underline{k}, \underline{x}, t)$ = dynamic component of transfer function model;
f_c = combined forecast;
f_i = forecast using method i;
k = number of forecasts available to combine;
$\underline{k}$ = vector of parameters associated with the transfer function component of the model;
m = particular season (month) of the year;
N = number of years of data;
N_t = noise component for transfer function model;
PAR$(p_1, p_2, \ldots, p_s)$ = periodic autoregressive model with seasonal order p_i $(i = 1, 2, \ldots, s)$;
P = order of the seasonal autoregressive operator;
p = order of the nonseasonal autoregressive operator;
p_i = order of the autoregressive operator for season i;
Q = order of the seasonal moving average operator;
q = order of the nonseasonal moving average operator;
s = number of seasons per year;
T = year;
t = discrete time;
w_i = weighting factor for the ith forecast;
$w_{i,j}$ = weighting factor for forecasting procedure i for the jth month;
$\underline{x}$ = vector of covariates at time t;
x_t = value of covariate series at time t;
$x_{t,i}$ = value of the ith covariate series at time t;
y_t = response variable at time t;

Z_t = time series observation at time t;
$Z_t^{(\lambda)}$ = transformed observation at time t;
α_{ij} = ith, jth element of inverse of forecast error covariance matrix;
δ = transfer function model parameter;
$\Theta(B^s)$ = seasonal moving average operator of order Q;
θ = nonseasonal moving average parameter;
$\theta(B)$ = nonseasonal moving average operator of order q;
λ = parameter of the Box-Cox transformation;
μ = mean of the series;
ν = number of previous forecasts used to calculate the combining weights;
$\nu_j(B)$ = jth transfer function;
$\mathbf{\Sigma}$ = forecast error covariance matrix;
Σ_{ij} = ith, jth element of $\mathbf{\Sigma}$;
σ_a^2 = variance of a;
$\Phi(B^s)$ = seasonal autoregressive operator of order P;
$\phi(B)$ = autoregressive operator of order p;
$\phi^{(m)}(B)$ = autoregressive operator of order p_m for season m;
ω = transfer function model parameter;
$(1 - B)^d$ = nonseasonal differencing operator of order d; and
$(1 - B^s)^D$ = seasonal differencing operator of order D.