Statistics 4657/9657: Assignment 2

Handout : September 24, 2008.
Due date: To be announced

1. 1.8.3.

   Possible example for part (c): If you use this example then work out the appropriate details.

   Consider $\Omega = \{1, 2, 3, 4\}$. Consider $\mathcal{F}$ to be the smallest σ-field containing $\{1\}$ and $\{2\}$. Consider $\mathcal{G}$ to be the smallest σ-field containing $\{3\}$ and $\{4\}$.

   If $\mathcal{F} \cup \mathcal{G}$ were a σ-field then show that it would need to contain $\{1, 3\} = \{1\} \cup \{3\}$.

   Does $\{1, 3\}$ belong to $\mathcal{F} \cup \mathcal{G}$?

2. (a) 1.8.11 (Boole's inequality). *Hint*: Use mathematical induction.

   (b) Use the Monotone property Lemma 5, page 7, and Boole's Inequality to show that Boole's Inequalilty also holds for $n = \infty$, that is show

   $$P(\cup_{i=1}^{\infty} A_i) \leq \sum_{i=1}^{\infty} P(A_i) \ .$$

3. 1.8.16.

   Remarks : B_n is a monotone decreasing sequence of events, so $B = \bigcap_{n \geq 1} B_n$ is a well defined event (use the definition of a sigma-field). In part (a) you need to show B as defined is equal to the set on the right hand side of the equation. Recall for two sets E_1 and E_2, they are equal if and only if $E_1 \subseteq E_1$ and $E_2 \subseteq E_1$. Part (b) requires similar calculations.

   For parts (c) and (d) you will need to work with the special properties and definition of A when the set B equals the set C.

4. 2.7.9

5. (Construction of Random Variables) In this problem U will be a uniform (0,1) valued random variable.

   F is the binomial(4,p) distribution function. Plot F and calculate F^{-1}. *Remark : F is a many to one function so you will need to think about what is its inverse.*

Write the definition of $F^{-1}(u)$, that is the rule of the function

$$F^{-1} : (0,1) \mapsto R \ .$$

Let U be a uniform. For $p = \frac{1}{2}$ verify that $X = F^{-1}(U)$ satisfies $P(X = k) = \binom{4}{k}\left(\frac{1}{2}\right)^4$ for $k = 0, 1, 2, 3, 4$.

Find the random variable $Y = F(X)$ and its distribution. Why is it not Unif(0,1)?

There are some conditions for which a r.v. $X \sim F$ (that is X has distribution function F) is such that $F(X) \sim$ Unif(0,1); what are they?

6. 3.7.2. Prove this for discrete random variables X and Y. *Remarks : (1) You will need to assume that E(Y) is finite. (2) This is a property of expectations and as a result of this Conditional Expectations will have certain properties.*

7. 3.11.1

Suggested problems

1. 2.1.4. *Remark*: This is the basis for constructing mixture distributions. These are often used in biostatistics, economics, actuarial sciences and other areas where *over dispersed* models are needed.

2. 2.1.5. These give various operations on distribution functions that produce other distribution functions.

3. 2.7.13. *Remark*: The set of distribution functions on the reals (and more general spaces) can be equipped with a distance and hence is a metric space. The study of weak convergence for stochastic processes is based on metrics for distributions in the appropriate setting.