Statistics 4657/9657: Assignment 3

Handout : October 15, 2008.
Due date: October 24, 2008, before 1:30 PM

1. X, Y_1 and Y_2 are random variables on a probability triple $(\Omega, \mathcal{F}, P)$. Let $\mathcal{G}$ be the σ-field generated by X. Suppose that Y_1, Y_2 have finite expectations.

   Below you may use properties of ordinary expectation and the definition of conditional expectation.

   (a) Let J, K be intervals on the real line. Suppose $a_1(X) = \mathrm{I}_J(X)$ and $a_2(X) = \mathrm{I}_K(X)$. Prove

   $$\mathrm{E}\left(a_1(X)Y_1 + a_2(X)Y_2 \mid \mathcal{G}\right) = a_1(X)\mathrm{E}(Y_1 \mid \mathcal{G}) + a_2(X)\mathrm{E}(Y_2 \mid \mathcal{G}) \ . \tag{1}$$

   (b) Let $J_1, J_2, \ldots, J_M$ be disjoint intervals and $K_1, K_2, \ldots, K_L$ be disjoint intervals on the real line. Let $\alpha_1, \ldots, \alpha_M$ and $\beta_1, \ldots, \beta_L$ be real numbers. Suppose $a_1(X) = \alpha_1 \mathrm{I}_{J_1}(X) + \ldots + \alpha_M \mathrm{I}_{J_M}(X)$ and $a_2(X) = \beta_1 \mathrm{I}_{K_1}(X) + \ldots + \beta_L \mathrm{I}_{K_L}(X)$.
   Prove that (1) holds.

2. 3.7.1 (b), (c), (d). Do these using the definition of conditional expectation.

3. 3.7.6

4. 5.6.3, 5.6.4

5. 5.7.2, 5.7.11 (these titled distributions are related to the Esscher transform used in mathematical finance and actuarial science).

6. 5.8.1