Statistics 3858b Assignment 3

Handout Feb 24, 2016; Due date: March (TBA) , 2016

These problems are all from the course text unless otherwise stated.

1. 9.11 : 4, 6, 17, 21, 23, 25, 26, 30 a, 37, 40
2. 10.9 : 19,
3. 11.6 : 7 , 10, 21 (part e means parametric bootstrap), 33

This next question shows the student how one can estimate parameters for a Markov chain. Both actuaries and financial modeling students work with some Markov processes and other time series statistical models. Here you will study the estimation for a 2 state Markov chain.

This problem is optional and is not required to be handed in.

4. A Markov chain with state space $\{0, 1\}$ has transition matrix

$$P = \begin{pmatrix} \alpha & 1-\alpha \\ 1-\beta & \beta \end{pmatrix} .$$

Suppose the starting value for the Markov chain is $X_0 = 0$, and that data given this starting position is $X_1, X_2, \ldots, X_n$. The initial state is $x_0 = 0$.

(a) Give the log likelihood function (conditional on $X_0 = 0$). Find the sufficient statistic; it will be a four dimensional statistic. You might read the next part of this question as a hint to find this sufficient statistic. Obtain a formula for the MLE of α and β.

(b) A data set of length $n + 1$, with $n = 200$ is obtained and has the following summary statistics

$$n_{0,0} = 95, n_{0,1} = 32, n_{1,0} = 32, n_{1,1} = 41$$

where $n_{l,k}$ counts the number of transitions from state l to state k in the data.

Obtain the maximum likelihood estimators of α and β.

(c) Using always the starting value or initial value of $X_0 = 0$, write R code to obtain the parametric bootstrap simulation approximation for the sampling distribution of the MLE.

You may make use of some relevant R functions given on the assignments page in your R code. These allow you to simulate the two state Markov chain and calculate the sample statistics needed.

(d) Using your code and the estimates above give the 95% confidence interval for α based on the parametric bootstrap approximation, with $M = 500$ bootstrap replicates.