Statistics 3858b Assignment 4

Handout March 18, 2016 ; Due date: March 30, 2016

These problems are all from the course text unless otherwise stated.

1. In the two sample normal case (see handout from class) consider the hypothesis test of

$$H_0 : \mu_X - \mu_Y = \delta_0 \text{ versus } H_A : \mu_X - \mu_Y > \delta_0$$

where δ_0 is a specific number.

   (a) Derive the GLR (generalized likelihood ratio) test.

   (b) Show the rejection region is of the form

$$R = \left\{ \mathbf{x}, \mathbf{y} : \frac{(\bar{x} - \bar{y} - \delta_0)}{\sqrt{S_n^2 \left(\frac{1}{n} + \frac{1}{m} \right)}} > c \right\}$$

   for some appropriate constant c.

2. Use the data from Problem 11.40 g. The field present data is a sample from one population distribution,say F, and field absent is a sample from a different population distribution, say G.

   (a) Analyze the data using the Mann Whitney non parametric method. Test the hypothesis $H_0 : F = G$ versus the alternative $H_A : F \neq G$. Test at level $\alpha = .05$. Do this both by using the function wlicox.test in R and by calculating the rank sum and using Table A8 from Rice.

   (b) Using only the field present data, that is a sample size $n = 10$, use the non parametric bootstrap method to obtain the 90% and the 95% confidence intervals for μ, the population mean.
   Base this confidence the studentized random variable

$$T = \frac{\sqrt{n}(\bar{X} - \mu)}{\sqrt{S^2}}$$

   where $\bar{X}$ is the sample mean r.v. and S^2 is the sample variance r.v. In your answer give the bootstrap quantiles for $0.025, 0.05, .95, .975$. Use the R package boot with $R = 3999$ replicates.

3. Use the data for Ozone group in 11.6, Question 35. It is a sample of size 22. Consider methods to obtain the confidence interval for $\mu = \mu(f)$, the population mean where F is the cdf and f pdf of the population distribution.

$$\mu(f) = \int_{-\infty}^{\infty} x f(x) dx \ .$$

This notation is intended to show that the population mean depends of the Base the confidence interval on

$$W = \frac{\sqrt{n}(\bar{X} - \mu(f))}{\sqrt{S^2}} \ . \tag{1}$$

Below all confidence intervals will be 95% confidence intervals so the quantiles needed are the .025 and .975 quantiles. For the bootstrap methods use 1999 bootstrap replicates.

   (a) Use the R package boot to obtain the .025 and .975 quantiles of (1).

   (b) Use the student's t distribution to obtain the .025 and .975 quantiles of (1).

   (c) Fit a parametric normal model to the data. Use the parametric bootstrap to obtain the .025 and .975 quantiles for (1).

   (d) Give the 95% confidence intervals for μ using the three methods above.

4. April 2012 exam question 3 (b) parts (ii), (iii).

5. 8.10.4 (e) [Bayes estimation]

6. (Bayes estimation) April 2012 exam, question 2.

Notice The final exam is on April 10, 7 PM. See the course web page for the room.