January 14, 2016 2:32 PM

$$(\Omega = [0,1],\ \overset{?}{\mathcal{F}},\ \overset{\lambda}{P})$$

$\mathcal{J}$ = \mathcal($\mathcal{J}$)
= { all intervals in [0,1] }

$(\cancel{\tfrac{1}{4}}, \tfrac{1}{2}) \cup (\tfrac{3}{4}, 1] \notin \mathcal{J}$

Ex 2.2.3
$\mathcal{J}$ is a semi-algebra

ie $\emptyset, \Omega = [0,1] \in \mathcal{J}$

$A \in \mathcal{J}$ then

$$A^c = \bigcup_{j=1}^{n} I_j, \quad I_1, \dots, I_n \in \mathcal{J}$$

$0 < a < b < 1$

$$[a,b]^c = [0,a) \cup (b,1]$$

ie A^c is the finite union of elements in $\mathcal{J}$

$\mathcal{B}_0$ = { all finite unions of elements of $\mathcal{J}$ }
= { all finite unions of intervals }

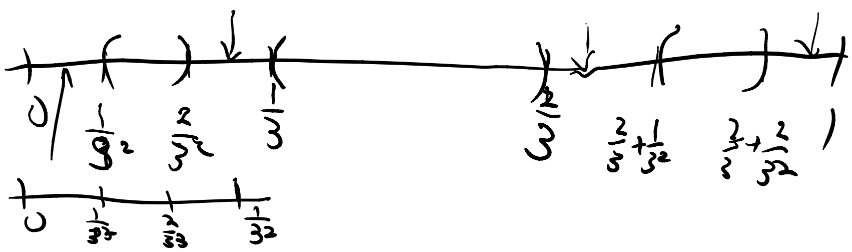
Ex 2.2.5 $\mathcal{F}$ σ-algebra if
(1) $\emptyset, \Omega \in \mathcal{F}$
(2) $A \in \mathcal{F}$ then $A^c \in \mathcal{F}$
(3) countable unions $\in \mathcal{F}$
if $A_1, A_2, \dots \in \mathcal{F}$ then
$\bigcup_{n=1}^{\infty} A_n \in \mathcal{F}$

{ b

$\mathcal{B}_1 = \{$ all countable unions of intervals $(a,b)\}$

See exercise 2.4.7

Cantor set K



Ex 2.4.7 (d), (e)

Remark

$x \in K$

$$x = \sum_{n=1}^{\infty} c_n \cdot 3^{-n} \qquad c_n = 0, 2$$

$c_1, c_2, c_3, \ldots$

$$d_1 = \begin{cases} 0 & \text{if } c_1 = 0 \\ 1 & \text{if } c_1 = 2 \end{cases} \quad \cdots \quad d_n = \begin{cases} 0 & \text{if } c_n = 0 \\ 1 & \text{if } c_n = 2 \end{cases}$$

$$y = \sum_{n=1}^{\infty} d_n \cdot 2^{-n}$$