January 21, 2016 2:32 PM

Suggested problems
2.4.7
Section 2.7
2.7.3 a,b
.4
.7
.18
.21

§2.4

$\mathcal{J}$ = set of all intervals in $[0,1]$

semialgebra $P(I)$ = length of I $= b - a$

I (a, b) $[\ \ ]$

Prop 2.4.2

$\mathcal{J}$, P obey (2.3.2) of extension theorem

$I_0 = \bigcup_{j=1}^{k} I_j$ disjoint union

a_1 $I_0 = \bigcup_{j=1}^{k} I_j$ b_k

$P(I_0) = b_k - a_1$

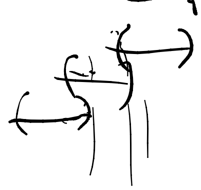
$\sum P(I_j) = \sum_{j=1}^{k} (b_j - a_j) = b_k - a_1$

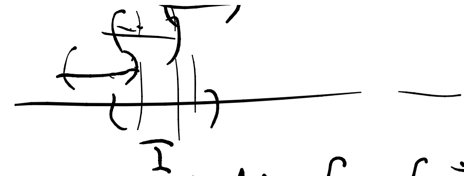
$b_j = a_{j+1}$

Exercise 2.4.3 verify (2.3.3)

(a) finite $I_1, \dots, I_n$

$\Rightarrow I \subseteq \bigcup_{i=1}^{n} I_i$





(2.3.3) holds for finite $I_1, \dots, I_n$

$I_1, I_2, \dots$ countable "cover"

$I \subseteq \bigcup_{i=1}^{\infty} I_i$

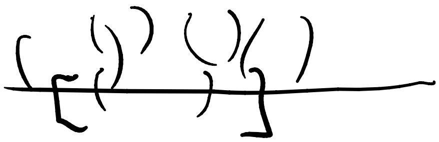
compactness

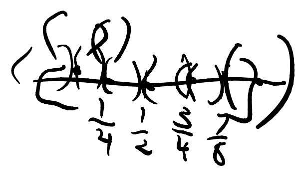
A ~~closed~~ compact set

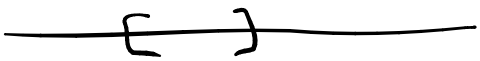
$\{ B_\alpha \mid \alpha \in \text{Index sets}, B_\alpha \text{ open} \}$ covers A

$\exists$ finite subcover, ie $B_{j_1}, \dots, B_{j_n}$

so that $A \subseteq \bigcup_{\ell=1}^{n} B_{j_\ell}$







I closed, $I_1, I_2, \dots$ open

cover I

$\exists$ finite $I_{j_1}, \dots, I_{j_n}$ which

cover I

ie $I \subseteq \bigcup_{\ell=1}^{n} I_{j_\ell}$

$$\therefore P(I) \leq \sum_{\ell=1}^{n} P(I_{j_\ell})$$

$$\leq \sum_{i=1}^{\infty} P(I_i) \quad /\!\!/$$

$$\| $$

$$\sup_{L} \sum_{\alpha \in L} P(I_\alpha)$$

L f.s.: fte subset of $\mathbb{N}$

$$\sup_{L} \sum_{\alpha \in L} \mu(I_\alpha)$$
L finite subset of $\mathbb{N}$

(c) $\qquad [\quad] \qquad ([\quad])$

$(\quad] = (a, b]$

$[a + n, b - n] \not\subset (a, b]$

Try to finish this argument.