

§ 2.6

Product Measures

$$(\Omega_1, \mathcal{F}_1, P_1) \quad (\Omega_2, \mathcal{F}_2, P_2)$$

prob triples

$$\mathcal{J} = \{ A \times B \mid A \in \mathcal{F}_1, B \in \mathcal{F}_2 \}$$

- it is a semi-algebra of
"measurable" rectangles

$\mathcal{J}$ subsets of $\Omega_1 \times \Omega_2 \equiv \Omega$

Define

$$P(A \times B) = P_1(A) P_2(B)$$

In order to find prob. triple

$$(\Omega, \mathcal{M}, P^*)$$

which extends P from $\mathcal{J}$ to $\mathcal{M}$

Need

① $\mathcal{J}$ semi algebra

② P satisfies either

Theorem 2.3.1

or Corollary 2.5.1

or Corollary 2.5.4

① $\mathcal{J}$ semi algebra

$$\phi = \underbrace{\phi_1}_{\subseteq \Omega_1} \times \underbrace{\Omega_2}_{\subseteq \Omega_2} = \Omega_1 \times \phi_2$$

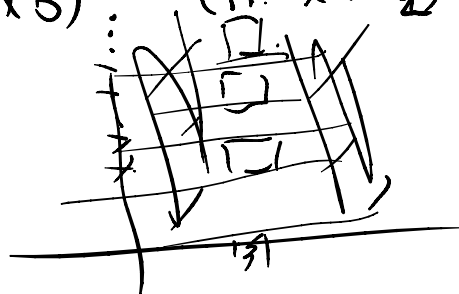
$$\phi = \phi_1 \times \phi_2 = \Omega_1 \times \phi_2$$

$\uparrow \subseteq \Omega_1 \quad \subseteq \Omega_1 \quad \subseteq \Omega_2$

$$\Omega = \Omega_1 \times \Omega_2$$

$(A \times B)^c$ finite union of elements of $\mathcal{G}$?

$$(A \times B)^c = (A^c \times \Omega_2) \cup (\Omega_1 \times B^c)$$

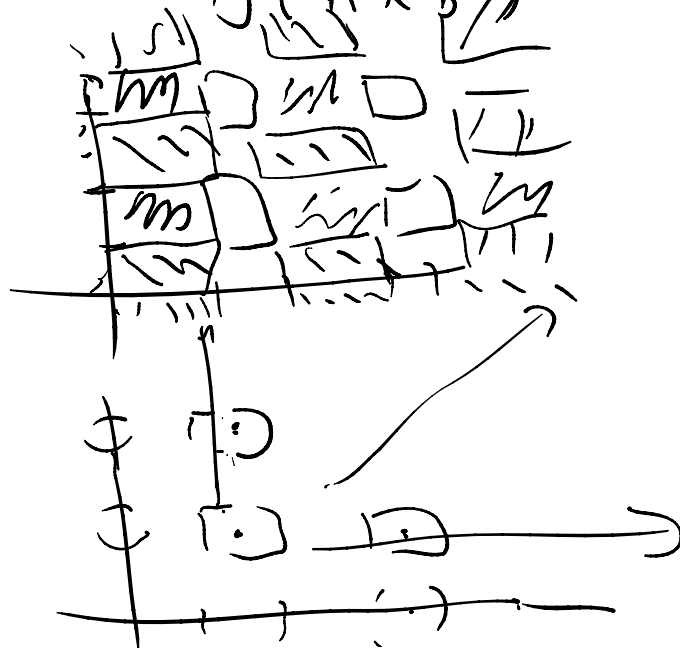


$$(A^c \times \Omega_2) \cup (\Omega_1 \times B^c)$$

$$(A \times B)^c = (A^c \times B^c)$$

$$\cup (A^c \times B)$$

$$\cup (A \times B^c)$$



R. shows $P, \mathcal{G}$ satisfy

R shows P, Q satisfy
Corollary 2.5.4 is (2.5.5)

Work Through Exercise 25.6
(shows (2.5.5) for finite disjoint
unions)

I am away Thurs Feb 4, 2016