February 25, 2016 2:22 PM

Possible times for an exam will be April 6, 7 or 8. Classes at UWO end April 6
A good time for an exam here is April 7 or 8 (Thursday or Friday).

§5 Inequalities and Convergence

5.2

$Z_n \to Z$ almost surely (a.s.) or almost everywhere (a.e.)

$Z_n(\omega) \to Z(\omega)$ $\qquad \omega \in \Omega$

$\{\omega \mid \lim_{n\to\infty} Z_n(\omega) = Z(\omega)\}$ $\qquad Z_n : \Omega \to \mathbb{R}$

$= \{\omega \mid Z_n(\omega) \to Z(\omega)\}$ $\qquad Z : \Omega \to \mathbb{R}$

$Z_n \to Z$ everywhere if $Z_n(\omega) \to Z(\omega)$ for all $\omega \in \Omega$

$Z_n \to Z$ a.s. if $P(\{\omega \mid \underbrace{Z_n(\omega) \to Z(\omega)}\}) = 1$

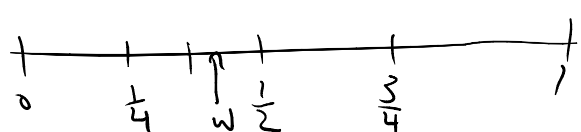
Convergence in probability

$Z_n \xrightarrow{P} Z$ iff $\forall \epsilon > 0$

$P(\ |Z_n - Z| > \epsilon) = P(\{\omega : \ |Z_n(\omega) - Z_n(\omega)| > \epsilon)$
$\to 0$

Convergence in prob is not the same as convergence a.s.

Eg $\Omega = [0,1]$, $\mathcal{F}$ = Borel sets, P uniform



Level 0 $\quad A_1 = [0,1]$

Level 1 $\quad A_2 = [0,\frac{1}{2}] \qquad A_3 = (\frac{1}{2},1]$

Level 2: $\quad A_4 = [0,\frac{1}{4}] \quad A_5 = (\frac{1}{4},\frac{1}{2}] \quad A_6 = (\frac{1}{2},\frac{3}{4}] \quad A_7 = (\frac{3}{4},1]$

Level k:

$[0,\frac{1}{2^k}] \quad (\frac{2}{2^k}, \frac{3+1}{2^k}], \ldots$

$(\frac{l}{2^k}, \frac{l+1}{2^{k}}], \ldots$

$Z_n(\omega) = I_{A_n}(\omega)$

$P(Z_n = 1) = P(A_n) = \frac{1}{2^k}$, $\quad 2^k \leq n < 2^{k+1}$

$$P(Z_n = 1) = P(A_n) = \frac{1}{2^k}, \quad \ell = [illegible]$$

$0 < \epsilon < 1$

$$\underbrace{P(\{\omega \mid |Z_n(\omega)| > \epsilon\})}_{E_n} = P(|Z_n - 0| > \epsilon) = P(Z_n = 1) = \frac{1}{2^k} \to 0$$

Z_n Bernoulli rv's, $Z_n \xrightarrow{P} 0$

Does Z_n converge a.s.?

$\omega \in [0, 1]$

$$\limsup_{n \to \infty} Z_n(\omega) = 1$$

$$\liminf_{n \to \infty} Z_n(\omega) = 0$$

Aside:

if $a_n \to a$. $\forall \epsilon > 0$

$\exists n_0(\epsilon) \ni \forall n \geq n_0, \; |a_n - a| \leq \epsilon$

$$\limsup_{n \to \infty} a_n = \lim_{n \to \infty} \underbrace{\sup_{m \geq n} a_m}_{\substack{n \geq n_0 \\ \sup(\cdot) \leq a + \epsilon}} \leq a + \epsilon$$

$$\liminf_{n \to \infty} a_n = \lim_{n \to \infty} \underbrace{\inf_{m \geq n} a_m} \geq a - \epsilon$$