February 11, 2016 2:38 PM

Can you hear our MIC?

Half good!

Chapter 4
Expectation

1) Simple positive r.v's

X is simple iff

$\text{Range}(X) = \{x \mid X(\omega) = x \text{ for some } \omega \in \Omega\}$

is a finite set

X is simple ~~and positive~~ (non-negative)

if $x \in \text{Range}(X) \geq 0$

$\text{Range}(X) = \{a_1, \dots, a_k\}$ a_j's distinct?

$X(\omega) = \sum_{j=1}^{k} a_j I_{A_j}(\omega)$

$A_j = \{\omega \mid X(\omega) = a_j\} = X^{-1}(\{a_j\})$

def (4.1.2) $E(X) = \sum_{j=1}^{k} a_j E(I_{A_j})$

$= \sum_{j=1}^{k} a_j P(A_j) \geq 0$

"Countable* simple" non-negative

$\text{Range}(X) = \{a_1, a_2, \dots\}$

is a countable set of values $a_j \geq 0$

$E(X) \equiv \sum_{j=1}^{\infty} a_j P(A_j)$

$A_j = X^{-1}(\{a_j\})$

from this definition (for simple non-neg r.v's)

( ) $E(c) = c$ ie $X(\omega) = c \ \forall \omega \in \Omega$

r.v's)

(1) $E(c) = c$ ie $X(\omega) = c \ \forall \omega \in \Omega$

(2) $E(I_A) = P(A)$ $A \in \mathcal{F}$

by taking

$$X = 0 \cdot I_{A^c} + 1 \cdot I_A$$

(3) X, Y simple ~~non-negative~~,

$a, b \in \mathbb{R}$

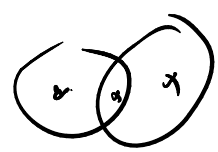
$$E(aX + bY) = aE(X) + bE(Y)$$

For r.v. on LHS

$aX + bY$

there are multiple representations

as simple r.v's

① $X = I_A$ 

$Y = I_B$

A, B, $A \cap B \neq \phi$

$$X + Y = I_{A \cap B^c} + 2I_{A \cap B} + I_{A^c \cap B}$$

$$= I_A + I_B$$

② $A \cap B = \emptyset$

Other properties

① $X \geq Y$ ie $\{\omega \mid X(\omega) \geq Y(\omega)\} = \Omega$

then for simple r.v's X, Y with

then for simple r.v's X, Y with

$X \geq Y$

$E(X) \geq E(Y)$

② $-|X| \leq X \leq |X|$

$$-E(|X|) = E(-|X|) \leq E(X)$$

$$E(X) \leq E(|X|)$$

$$\therefore \quad |E(X)| \leq E(|X|)$$

③ if X, Y are independent simple

$$E(XY) = E(X)E(Y)$$

$$XY = \left(\sum_{j=1}^{k} a_j I_{A_j}\right)\left(\sum_{l=1}^{m} b_l I_{B_l}\right)$$

$$= \sum_{j=1}^{k}\sum_{l=1}^{m} a_j b_l I_{A_j} \cdot I_{B_l}$$

$$= \sum_{j=1}^{k}\sum_{l=1}^{m} a_j b_l I_{A_j \cap B_l}$$

$$I_A \cdot I_B = I_{A \cap B}$$

$$E(XY) = \sum_{j=1}^{k}\sum_{l=1}^{m} a_j b_l E\left(I_{A_j} \cdot I_{B_l}\right)$$

use indep. of X, Y

to conclude A_j, B_l independent

$$= \sum_{j=1}^{k}\sum_{l=1}^{m} a_j \cdot P(A_j) \cdot b_l P(B_l)$$

$$= E(X)E(Y)$$