March 3, 2016 2:33 PM

Some problems from the text

4.5.3
4.5.12 Hint (a) need to change dyadic form to something else; eg triadic
4.5.15

5.5.5
5.5.8 (a), (b)
5.5.9
5.5.13
6.3.5

Re: 4.5.12

$\psi_n(x) = \min(n, 2^{-n}[2^n x])$

$x \geq 0$

$X_n = \psi_n(X) \uparrow$

Re: 5.5.13

$\varepsilon_i, \ i \geq -r$ iid

$X_i = a_1\varepsilon_i + a_2\varepsilon_{i-1} + \dots + a_r\varepsilon_{i-r+1}$

$= f(\varepsilon_i, \dots, \varepsilon_{i-r+1})$

§5 2^{nd} Version of SLLN

① + ② $X_i \geq 0$ WLOG:

Need to consider $X_i = X_i^+ - X_i^-$

$X_i^+, X_i^- \geq 0$.

IF we can prove SLLN for these ~~positive~~ non-negative (≥ 0) r.v.'s, then

$\exists A^+, A^-$, $P(A^+) = P(A^-) = 1$

$\Rightarrow \forall \omega \in A^+$

$\frac{1}{n}\sum_{i=1}^n X_i^+(\omega) \to m^+$

and

$\forall \omega \in A^-$, $\frac{1}{n}\sum_{i=1}^n X_i^-(\omega) \to m^-$

If $\omega \in A^+ \cap A^-$, then

$\frac{1}{n}\sum_{i=1}^n X_i(\omega) = \frac{1}{n}\sum_{i=1}^n X_i^+(\omega) - \frac{1}{n}\sum_{i=1}^n X_i^-(\omega)$

$\to m^+ - m^- = E(X) = m$

$P((A^+ \cap A^-)^c)$

$= P((A^+)^c \cup (A^-)^c)$

$\leq P((A^+)^c) + P((A^-)^c) = 0 + 0 = 0$

Truncation WLOG $X_i \geq 0$

(i)

Truncation WLOG $X_i \geq 0$

$Y_i = X_i I(X_i \leq i)$

(5) Show $P(X_i \neq Y_i \text{ i.o.}) = 0$

Method to Prove:

1st Borel Cantelli Lemma

$$\sum_{i=1}^{\infty} P(X_i \neq Y_i)$$

$X_i = Y_i$ if $0 \leq X_i \leq i$

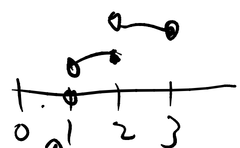
$X_i \neq Y_i$ if $X_i > i$

$$= \sum_{i=1}^{\infty} P(X_i > i)$$

$$= \sum_{i=1}^{\infty} P(X_1 > i)$$

$$= \sum_{i=1}^{\infty} \left\{ \sum_{j=i}^{\infty} P(j < X_1 \leq j+1) \right\}$$

$$= \sum_{j=1}^{\infty} \left\{ \sum_{i=1}^{j} P(j < X_1 \leq j+1) \right\}$$



$$\left(= \sum_{j=1}^{\infty} E(h_j(X_1)) \right)$$

$h(x) =$

$h(x) \leq x$ on $[0, j+1]$

$$= \sum_{j=1}^{\infty} E\left(\sum_{i=1}^{j} I(j < X_1 \leq j+1) \right)$$

$$= \sum_{j=1}^{\infty} j\, E(I(j < X_1 \leq j+1))$$

$$\leq \sum_{j=1}^{\infty} E(X_1 I(j < X_1 \leq j+1))$$

$$= E\left(\sum_{j=1}^{\infty} X_1 I(j < X_1 \leq j+1) \right)$$

(see p 48 just above Prop 4.2.9 }, line (4.2.8))

$$= E(X_1 I(X_1 > 1))$$

$$\leq E(X_1) = m < \infty$$

So now we need to look at step 4

ie show $\frac{1}{n} \sum_{i=1}^{n} Y_i \to m$ a.s.

$\bar{Y}_n$