Derivation of $n \geq 2\ p$ in the Champernowne Equation

Introduction

Champernowne (1948, eqn 3.5) showed that

$$z' \Gamma_n^{-1} z = \beta' D\ \beta / \sigma_a^2, \qquad (1)$$

where $\beta = (-1, \phi_1, \ \ldots, \ \phi_p)$ and D is the $(p+1)$-by-$(p+1)$ matrix with $(i, \ j)$-entry

$$D_{i,j} = D_{j,i} = z_i z_j + \ldots + z_{n+1-i} z_{n+1-j} \qquad (2)$$

Just below eqn (3.2), Champernowne (1948) indicates that $n > 2\ p$. Close examination of the derivation given in Champernowne (1948) and more elegantly by Box, Jenkins and Reinsel (1994, A7) indicates that a necessary and sufficient condition for (1) above to hold is that $n \geq 2\ p$.

Computation of D

```
GD[p_, n_, z_] := Module[{},
  D = Array[0 &, {p + 1, p + 1}];
  Do[
   D[[i, j]] = Sum[z[[i + k]] z[[j + k]],
     {k, 0, n + 1 - i - j}], {i, p + 1}, {j, p + 1}];
  D]
```

```
z = Table[Subscript[z, k], {k, 4}];
```

```
GD[2, 4, z]
```

$\{\{z_1^2 + z_2^2 + z_3^2 + z_4^2,\ z_1 z_2 + z_2 z_3 + z_3 z_4,\ z_1 z_3 + z_2 z_4\},$
$\{z_1 z_2 + z_2 z_3 + z_3 z_4,\ z_2^2 + z_3^2,\ z_2 z_3\},\ \{z_1 z_3 + z_2 z_4,\ z_2 z_3,\ 0\}\}$

```
<< FitAR.m
```

Updated January 7, 2007.Loaded FitAR

■ Exercise. Take p=2 and n=5.

```
z = Range[5]
```

```
{1, 2, 3, 4, 5}
```

```
GD[2, 5, z]
```

```
{{55, 40, 26}, {40, 29, 18}, {26, 18, 9}}
```

```
β = {-1, 1.6, -0.64};
β.D.β
```

```
1.3424
```

```
G = Toeplitz[TacvfAR[{1.6, -0.64}, 4]];
z.Inverse[G].z
```

```
1.3424
```

■ Exercise. Take p=2 and n=4.

```
z = Range[4]
```

```
{1, 2, 3, 4}
```

```
GD[2, 4, z]
```

```
{{30, 20, 11}, {20, 13, 6}, {11, 6, 0}}
```

```
β = {-1, 1.6, -0.64};
β.D.β
```

```
1.072
```

```
G = Toeplitz[TacvfAR[{1.6, -0.64}, 3]];
z.Inverse[G].z
```

```
1.072
```

■ Exercise. Take p=2 and n=3.

```
z = Range[3]

{1, 2, 3}

GD[2, 3, z]

{{14, 8, 3}, {8, 4, 0}, {3, 0, 0}}

β = {-1, 1.6, -0.64};

β.D.β

2.48

G = Toeplitz[TacvfAR[{1.6, -0.64}, 2]];

z.Inverse[G].z

0.8416
```